Department of Electrical Engineering
Sessional Assignment
Date: 01/06/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:		**Total Marks:**	20

Student Details

Name: ABDUL BASIT	**Student ID: 13684**

Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$ To the input $x(n) = 4^n u(n)$.	Marks 6
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$$	
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	Marks 6
	(b)	Determine the partial fraction expansion of the following proper function $$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$	
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $\left\|H(\frac{\pi}{4})\right\|^2 = \frac{1}{2}$.	Marks 4

	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.	
Q 4	(c)	A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & otherwise \end{cases}$$ Determine the N- point DFT of this sequence for $N \ge L$	Marks 4
	(d)	Compute the DFT of the four-point sequence $$x(n) = (0\ \ 1\ \ 2\ \ 3)$$	

Digital Signal Processing Sessional Assignment

Name:- Abdul-Basit

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Q (a) Determine the response $y(n)$, $n \geq 0$, of the system described by the second-order difference equation

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

to the input $x(n) = 4^n u(n)$.

Solution:-

Consider the difference equation

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1) \rightarrow (1)$$

The homogenous equation of the system is

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

The characteristic equation of the system is

$\lambda - 3\lambda^{-1} - 4\lambda^{-2} = 0$

$\lambda^2 - 3\lambda - 4 = 0$

Determine the root of the characteristic equation

$\lambda^{-2} - 4\lambda + \lambda - 4 = 0$

$\lambda(\lambda - 4) + 1(\lambda - 4) = 0$

$(\lambda - 4)(\lambda + 1) = 0$

$\lambda = -1, 4$

The homogenous solution is,

$y_h(n) = C_1(-1)^n u(n) + C_2(4)^n u(n).$

Since 4 is a characteristic root and the excitation is

$x(n) = 4^n u(n)$

we assume a particular solution of the form

$y_p(n) = Kn4^n u(n).$

Then

$$Kn4^n u(n) - 3K(n-1)4^{n-1}u(n-1) - 4K(n-2)4^{n-2}u(n-2)$$
$$= 4^n u(n) + 2(4)^{n-1} u(n-1).$$

For $n=2$

$$K(32-12) = 4^2 + 8 = 24 \rightarrow K \doteq \frac{6}{5}.$$

The total solution is

$$y(n) = y_p(n) + y_h(n)$$
$$= \left[\frac{6}{5} n4^n + c_1 4^n + c_2(-1)^n\right] u(n).$$

To solve for c_1 and c_2, we assume that $y(-1) = y(-2) = 0$. Then

$y(0) = 1$ and

$y(1) = 3y(0) + 4 + 2 = 9.$

Hence $c_1 + c_2 = 1$ and

$$\frac{24}{5} + 4c_1 - c_2 = 9$$
$$4c_1 - c_2 = \frac{21}{5}.$$

There fore,

$$C_1 = \frac{26}{25} \quad \text{and} \quad C_2 = -\frac{1}{25}$$

The total solution is

$$y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25} (-1)^n\right] u(n)$$

(b) Determine the impulse response and unit step response of the systems describe by the difference equation.

$$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n).$$

Sol

Consider the difference equation

$$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$$

$$y(n) - 0.6y(n-1) + 0.08y(n-2) = x(n).$$

To obtain the homogenous equation set input

$$x(n) = 0$$

$$y(n) - 0.6y(n-1) + 0.08y(n-2) = 0.$$

Determine the solution to the homogenous equation

$$y_h(n) = \lambda^n$$

Substitute the solution obtained in the homogenous equation

$\lambda^{n} - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$

$\lambda^{n-2}(\lambda^{2} - 0.6\lambda + 0.08) = 0.$

$\lambda^{2} - 0.6\lambda + 0.08 = 0$

$(\lambda - 0.2)(\lambda - 0.4) = 0$

Therefore, the roots are

$\lambda_1 = 0.2, \ \lambda_2 = 0.4$

Thus, the general form of the Solution to the homogenous equation is,

$y_h(n) = C_1(\lambda_1)^n + C_2(\lambda_2)^n$

$y(n) = C_1(0.2)^n + C_2(0.4)^n \dots (1)$

$\lambda = 0.2, \ \lambda = 0.4$ hence

$y_h(n) = C_1 \frac{1^n}{5} + C_2 \frac{2^n}{5}.$

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(7)

With $x(n) = \delta(n)$, the initial condition are

$$y(0) = 1,$$

$$y(1) - 0.6y(0) = 0$$

$$y(1) = 0.6$$

Hence $c_1 + c_2 = 1$ and

$$\frac{1}{5}c_1 + \frac{2}{5} = 0.6$$

$$\Rightarrow c_1 = -1, \quad c_2 = 3.$$

Therefore $h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right]u(n).$

The step response is

$$s(n) = \sum_{k=0}^{n} h(n-k), \; n > 0$$

$$= \sum_{k=0}^{n} \left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right].$$

$$= \left\{\frac{1}{0.12}\left[\frac{2}{5}^{n+1} - 1\right] - \frac{1}{0.16}\left[\left(\frac{1}{5}\right)^{n+1} - 1\right]\right\}u(n).$$

Q2(a) Determine the causal signal $x(n)$ having the Z-transform.

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}.$$

(Hint :- Take inverse Z-transform using partial fraction method).

Solution:-

The Z-transform is,

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

The expression is written as.

$$X(z) = \frac{1}{\left(1-\frac{2}{z}\right)\left(1-\frac{1}{z}\right)^2}.$$

$$= \frac{1}{\left(\frac{z-2}{z}\right)\left(\frac{z-1}{z}\right)^2}$$

$$= \frac{1}{\frac{(z-2)(z-1)^2}{z^3}}$$

$$= \frac{z^3}{(z-2)(z-1)^2} \text{ ----- } (1).$$

$X(z)$ has a simple pole at $p_1 = 2$ and a double $p_2 = p_3 = 1$. In such a case the appropiate partial-fraction expansion is.

$$X(z) = \frac{z^3}{(z-2)(z-1)^2} = \frac{A_1}{z-2} + \frac{A_2}{z-1} + \frac{A_3}{(z-1)^2}.$$

The problem is to determine the coefficient A_1, A_2 and A_3.

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We proceed as in the case of distinct pole To determine A_1 we multiply both side of by $(z-2)$ and evaluate the result $z=-2$

$$(z-2)X(z) = A_1 + \frac{z-2}{z-1}A_2 + \frac{z-2}{(z-1)^2}A_3.$$

which we evaluated at $z=2$

$$A_1 = \left.\frac{(z-2)X(z)}{z}\right|_{z=2}$$

$$A_1 = 4$$

$$A_2 = A_1 + \frac{z-2}{z-1}$$

$$A_2 = -3.$$

$$A_3 = A_1 + \frac{z-2}{z-1}A_2$$

$$= -1.$$

Hence $x(n) = [4(2)^n - 3 - n]u(n)$

(b) Determine the partial fraction expansion of the following proper function.

$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}.$$

Sol First we eliminate the negative power by multiplying both numerator and de-nominator by z^2 thus.

$$X(z) = \frac{z^2}{z^2 - 1.5z + 0.5}.$$

The poles $X(z)$ are $p_1 = 1$ and $p_2 = 0.5$

Consequently, the expansion of the form

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A_1}{z-1} + \frac{A_2}{z-0.5}.$$

A very simple method to determine A_1 and A_2 is to multiply the equation by the denominator term $(z-1)(z-0.5)$ thus we obtain

$$z = (z-0.5)A_1 + (z-1)A_2. \rightarrow ①$$

Now if we set $z = p_1 = 1$ in eq ① we eliminate the term involving A_2. Hence.

$$1 = (1-0.5)A_1$$

Thus we obtain the result $A_1 = 2$ Next we return eq ① and $z = p_2 = 0.5$ thus eleminating the term involving A_1 so we have

$$0.5 = (0.5-1)A_2.$$

and hence $A_2 = -1$, Therefore the result of the partial fraction expansion is.

$$\frac{X(z)}{z} = \frac{2}{z-1} - \frac{1}{z-0.5} \quad \text{Ans.}$$

Question No 3(a).

Solution

At $w=0$ we have

$$H(0) = \frac{b_0}{(1-P)^2} = 1$$

hence

$$b_0 = (1-P)^2$$

At $w = \pi/4$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{(1-P\cos(\pi/4)+jP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2}+jP\sqrt{2})^2}$$

Hence

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$$\frac{(1-p)^4}{[(1-p/\sqrt{2})^2+p^2/2]^2} = \frac{1}{2}.$$

or equivalently

$$\sqrt{2}(1-p)^2 = 1+p^2-\sqrt{2}p$$

The value of $p=0.32$ satisfies this equation. Consequently, the system function for the desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}.$$

Question No 3(b):

Sol

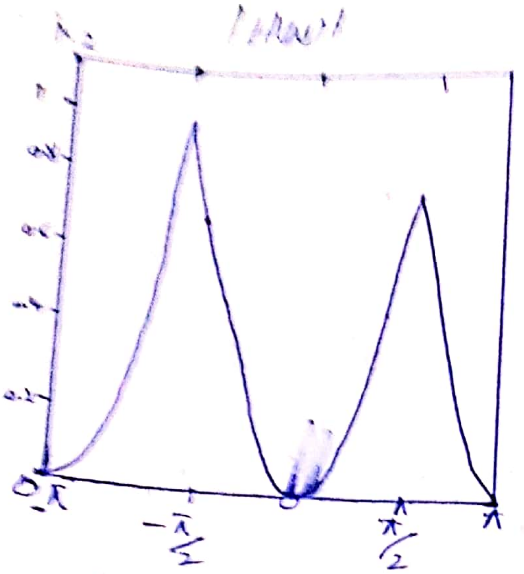
Clearly, the filter must have poles at

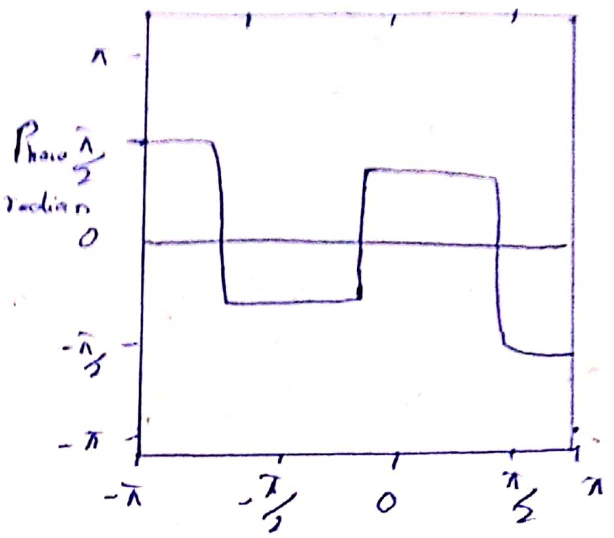
$$P_{1,2} = re^{\pm j\pi/2}.$$

and zeros at $z=1$ and $z=-1$, consequently, the system function is.

$$H(z) = \frac{G(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}.$$





The gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$ thus we have

$$H\left(\frac{\pi}{2}\right) = G\,\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}.$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have.

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4}\;\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = \frac{1}{2}.$$

or equivalently

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ satisfies this equation. Therefore, the system function for the desired filter is

$$H(z) = 0.15\frac{1+z^{-2}}{1+0.7z^{-2}}$$

Question 4 (a).

Sol

The fourier transform of this sequence is

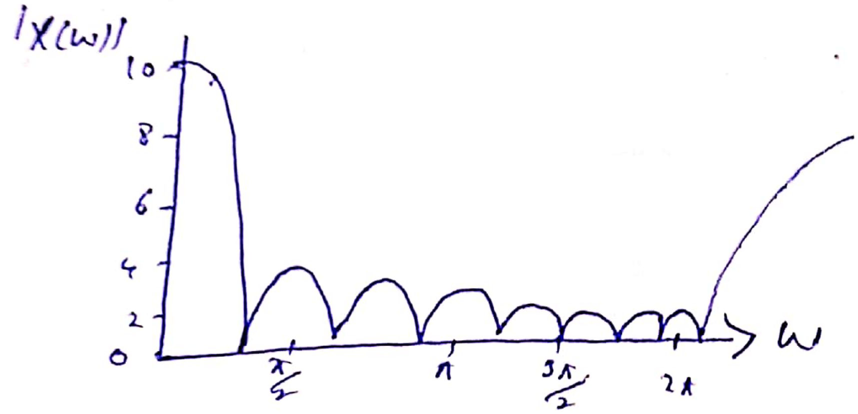
$$X(\omega) = \sum_{n=0}^{L-1} x(n)e^{-j\omega n}$$

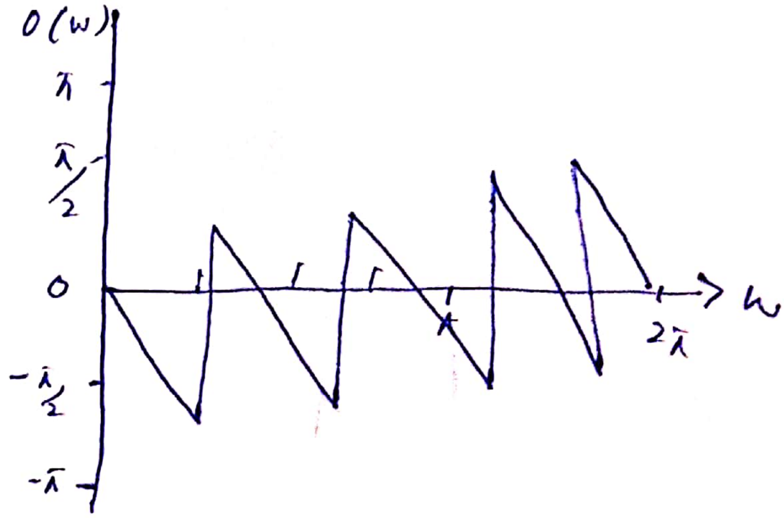
$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}} = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $X(\omega)$ are illustration for $L=10$. The N-point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally spaced frequencies $\omega_k = 2\pi K/N$, $K = 0, 1, \dots N-1$ Hence,

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{-j2\pi K/N}}, \quad K = 0, 1, \dots, N-1$$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}.$$





If N is selected such that N = L, then the DFT become

$$X(k) = \begin{cases} L, & k = 0 \\ 0, & k = 1, 2, \ldots, L-1 \end{cases}$$

Thus there is only one non zero value in DFT. This is apparent from observation of $X(\omega)$ since $X(\omega) = 0$ at the frequencies $\omega_k = 2\pi k/L$ $k \neq 0$. The reader should verify that $x(n)$ can be recovered from $X(k)$ by performing an L-point DFT.

Question NO 4(b).

Sol

The first step is to determine the matrix W_4. By exploiting the periodicity property of W_4, and the symmetry property

$$W_N^{k+N/2} = -W_N^k$$

the matrix W_4 may be expressed as.

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^4 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Then

$$Y_4 = W_4 Y_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

The DFT of Y_4 may be determined by Conjugating the element in W_4 to obtain W_4^{+} and then applying the formula.