ASSIGNMENT NO # 03

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Section: A

Subject: Differential Equations

Topic: Applications of Partial Differential Equations

Applications of PDF's:

Partial Differential Equation play important role in our real life, Mathematical Modeling, here we discuss some well-known equations and its Applications.

1).Heat Equation:

Statement of Heat Equation:-

For the function $u(x, y, z, t)$ of three spatial variables (x, y, z) and time variable t the heat equation is :

$$\frac{\partial u}{\partial t} = \alpha\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \qquad (1)$$

Where α = diffusivity of the medium.

Eq(1) can also be written in the form

$$\dot{u} = \alpha \nabla^2 u \qquad (2)$$

Where $\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$ is called Laplace Operator

This Eq(1 or 2) describes the flow of heat in a homogenous and isotropic medium, with $u(x, y, z, t)$ being temperature at the point (x, y, z) and time t .

(Reference: www.en.wikipedia.org)

Applications of Heat Equation:

- Water is used in Automobile Engine as a heat absorb due to high specific heat capacity.
- The hot water bottle is used in a hospital to release stomach pain because of the specific heat capacity of water is more and is slowly cools down.

(Reference: www.quora.com)

2).Wave Equation:

Statement of the Wave Equation:

The wave equation is a PDE that may constrain some scalar functions $u = u(x_1, x_2, x_3, \ldots, x_n; t)$ of a time variable t and one or more spatial variables i,e: $x_1, x_2, x_3, \ldots, x_n$.

The quantity u may be the pressure/displacement in a liquid/gas, along some specific direction, of the particle of a vibrating solid way form :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \qquad (1)$$

$$\ddot{u} = \alpha \nabla^2 u \qquad (2)$$

Where c = Non-negative real co-efficient.

In Eq(2) $\ddot{u}$ denotes duble time derivatives

Where ∇ is the nabla Operator

And $\nabla . \nabla = \nabla^2$ is called Laplacian Operator

By comparing Eq(1) and Eq(2):

$$\ddot{u} = \frac{\partial^2 u}{\partial t^2}, \; \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_1^2} +, \ldots, + \frac{\partial^2}{\partial x_n^2}.$$

(Reference:www.en.wikipedia.org)

Applications of Wave Equation:

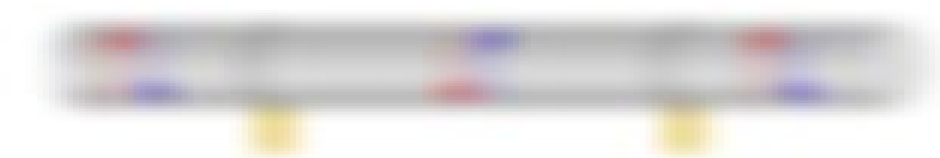
- The wave equation governs a wide range of phenomena, including
- Gravitational Waves
- Light Waves
- Sound Waves
- Oscillation of the strings in a string theory

Applications of Wave Equation in Civil Engineering:

The vibrating string problem (Wave Equation) related to several civil engineering Applications

- Vibrating Beam problem is a 4^{th} order PDE.

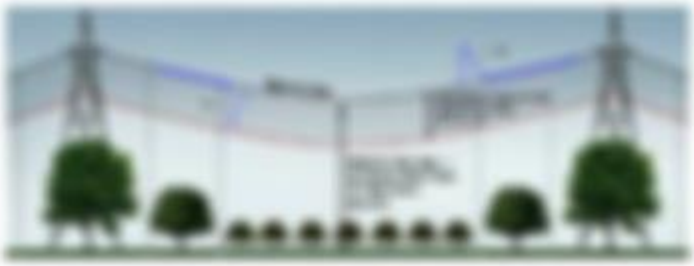
- Torsional vibration of a Rod (e.g: in machines deriven by electric motors)



- Vibration of the Earth’s surface and/or structures (e.g: due to an earthquake).

- Flood and Tsunami wave.

- Swaying of overhead transmission lines (e.g.: Phone, Power & Fiber Optic Cable).



3). Laplace Equations:

Statement of the Laplace Equation:

In mathematics, Laplace's equation is a second-order partial differential equation named after Pierre-Simon Laplace who first studied its properties. This is often written as

$$\nabla^2 f = 0 \text{ or } \Delta f = 0 \qquad (1)$$

where $\Delta = \nabla . \nabla = \nabla^2$ is the Laplace operator, ∇ is the divergence operator (also symbolized "div"), ∇ is the gradient operator (also symbolized "grad"), and $f(x, y, z)$ is a twice-differentiable real-valued function. The Laplace operator therefore maps a scalar function to another scalar function.

If the right-hand side is specified as a given function, $h(x, y, z)$, we have

$$\Delta f = h(x, y, z) \qquad (2)$$

Eq. (2) is called Poisson's equation, a generalization of Laplace's equation. Laplace's equation and Poisson's equation are the simplest examples of elliptic partial differential equations.

Laplace's equation is also a special case of the Helmholtz equation.

(Reference: https://en.wikipedia.org/)

Applications of Laplace Equation:

1). Laplace Equation have important in many fields of science, notably the fields of

- electromagnetism
- Astronomy
- fluid dynamics

Because they can be used to accurately describe the behavior of electric, gravitational, and fluid potentials.

2). Laplace equation one can make a simple mathematical model to relate between input and output.

(Reference: https://www.quora.com)

4). Naiver-Stoke Equation:

Statement of the Naiver-Stoke Equation:

In Mathematics, the **Naiver–Stokes equations** are a set of differential equations which describe the motion of viscous fluid substances, named after French engineer and physicist Claude-Louis Naiver and Anglo-Irish physicist and mathematician George Gabriel Stokes.

These balance equations arise from applying Isaac Newton's second law to fluid motion, together with the assumption that the stress in the fluid is the sum of a diffusing viscous term (proportional to the gradient of velocity) and a pressure term—hence describing *viscous flow*

The Naiver–Stokes momentum equation can be derived as a particular form of the Cauchy momentum equation, whose general convective form is

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \nabla.\tau + \rho g \qquad (1)$$

Where

- $\frac{D}{Dt}$ is the material derivative , defined as $\frac{\partial}{\partial t} + \mathbf{u}.\nabla$
- ρ is the density
- $\mathbf{u}$ is the flow velocity
- ∇ is the divergence
- p is the pressure
- t is the time
- τ is the deviation stress tensor, which has order two.
- g represents body acceleration acting on the continuum (e.g.: Gravity, Internal Acceleration etc.)

(Reference: https://en.wikipedia.org)

Applications of the Naiver stoke Equations:

The N-S equations (note the plural) describe fluid flow. They are used for

- Weather forecast
- Aero plane design and testing
- Ship design and testing
- Determining currents in rivers
- Calculating drainage systems
- Design of harbors and dykes

- Design of combustion and jet motors
- Design of windmills

and any design where the inertia and viscosity of the fluid plays a role. So only ground water flow where velocities are typically a few inches a day and the driving force is the hydrostatic pressure, is not modeled by the NS equations.

(Reference: https://www.quora.com)