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Section B

Department (SE) @

Semester 2nd

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(1)

(Answer No 1)

part (i)

$$ID = 18284$$

$$A = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$A_{ij} = (-1)^{i+j} m_{ij}$$

$$\text{Adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A_{11} = (-1)^{1+1} m_{11} = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}$$

$$A_{11} = m_{11} = 6 - 1$$

$$\boxed{A_{11} = 5}$$

$$A_{12} = (-1)^{1+2} m_{12} = \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$$

$$= -0 m_{12} = -(4-3) \\ = -1$$

$$\boxed{A_{12} = -1}$$

$$A_{13} = (-1)^{1+3} m_{13} = \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix}$$

$$A_{13} = m_{13} = 2-9 \\ = -7$$

$$\boxed{A_{13} = -7}$$

$$A_{21} = -m_{21} = \begin{vmatrix} 2 & 6 \\ 1 & 2 \end{vmatrix}$$

$$= -(4-6)$$

$$= -(-2)$$

$$A_{21} = 2$$

$$A_{22} = M_{22} = \begin{vmatrix} 1 & 6 \\ 3 & 2 \end{vmatrix}$$

$$= 2 - 18$$

$$A_{22} = -16$$

$$A_{23} = -M_{23} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$$

$$= -(1 - 6)$$

$$= -(-5)$$

$$A_{23} = 5$$

$$A_{31} = M_{31} = \begin{vmatrix} 2 & 6 \\ 3 & 1 \end{vmatrix}$$

$$= 2 - 18$$

$$A_{31} = -16$$

$$A_{32} = -M_{32} = \begin{vmatrix} 1 & 6 \\ 2 & 1 \end{vmatrix}$$

$$= -(1 - 12)$$

$$= -(-11)$$

$$A_{32} = 11$$

$$A_{33} = M_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$$

$$= 3 - 4$$

$$= -1$$

$$A_{33} = -1$$

2

So

$$\text{Adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

putting values-

$$\text{Adj } A = \begin{bmatrix} 4 & -1 & -7 \\ 2 & -16 & 5 \\ -16 & 11 & -1 \end{bmatrix}$$

(part B)

$$B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\begin{aligned} a_{11} = m_{11} &= -8 - (-16) \\ &= -8 + 16 \\ &= 8 \end{aligned}$$

$$\boxed{a_{11} = 8}$$

$$\begin{aligned} a_{12} = -m_{12} &= -(18 - 40) \\ &= 24 \end{aligned}$$

$$\boxed{a_{12} = 24}$$

$$\begin{aligned} a_{13} = m_{13} &= -4 - (-5) \\ &= -4 + 5 \\ &= 1 \end{aligned}$$

$$\boxed{a_{13} = 1}$$

$$\begin{aligned} a_{21} = -m_{21} &= -(32 + 10) \\ &= -42 \end{aligned}$$

$$\boxed{a_{21} = -42}$$

$$\begin{aligned} a_{22} = m_{22} &= 24 - 25 \\ &= -1 \end{aligned}$$

$$\boxed{a_{22} = -1}$$

$$\begin{aligned} a_{23} &= -m_{23} = -(-6-20) \\ &= -(-26) \\ &= 26 \end{aligned}$$

$$a_{23} = 26$$

~~21~~

$$\begin{aligned} a_{31} &= m_{31} = 32 + 5 \\ &= 37 \end{aligned}$$

$$a_{31} = 37$$

$$\begin{aligned} a_{32} &= -m_{32} = -(28 - 24 - 10) \\ &= -(14) \\ &= -14 \end{aligned}$$

$$a_{32} = -14$$

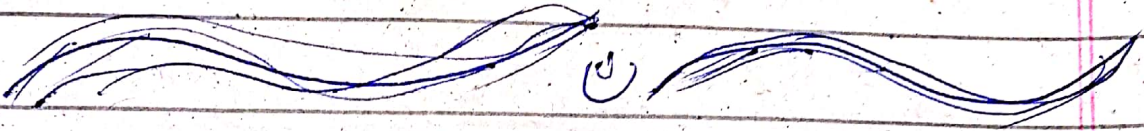
$$\begin{aligned} a_{33} &= m_{33} = -3 - 8 \\ &= -11 \end{aligned}$$

$$a_{33} = -11$$

So



$$\text{Adj } A = \begin{bmatrix} 8 & 24 & 1 \\ -42 & -1 & 26 \\ 37 & -14 & -11 \end{bmatrix}$$



Answer No 2)

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

Now finding  $A_{21}$

$$A_{21} = \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix}$$

$$= -4 + 9$$

$$\boxed{A_{21} = 5}$$

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Now finding  $A_{31}$

$$A_{31} = \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix}$$

$$= -2 - 9$$

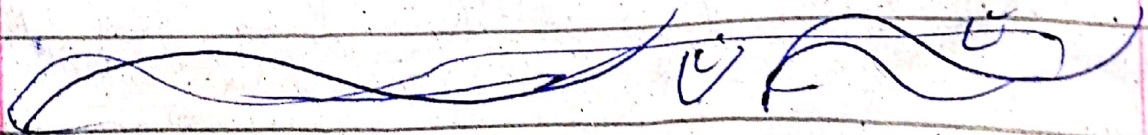
$$\boxed{A_{31} = -11}$$

$$A_{33} = \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix}$$

$$= 3 - 4$$

$$= -1$$

$$\boxed{A_{33} = -1}$$



(Answer No 3)

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$$

and  $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now for finding Eigenvalues  
we use this theorem.

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ -1 & 1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda) \left( (3-\lambda)(2-\lambda) - 2 \right) - 1 \left( (2-\lambda) + 2 \right) + 1 \left( 1 + (3-\lambda) \right) = 0$$

$$\bullet (2-\lambda) (6 - 3\lambda - 2\lambda + \lambda^2 - 2) - (2-\lambda + 2) + 1(1+3-\lambda) = 0$$

$$\bullet (2-\lambda) (4 - 5\lambda + \lambda^2) - (4-\lambda) + (4-\lambda) = 0$$

$$\bullet (2-\lambda) (4 - 5\lambda + \lambda^2) = 0$$

$$2-\lambda = 0$$

$$-\lambda = -2$$

$$\boxed{\lambda = 2}$$

$$\bullet \lambda^2 - 5\lambda + 4 = 0$$

$$\lambda^2 - \lambda - 4\lambda + 4 = 0$$

$$\lambda(\lambda-1) - 4(\lambda-1) = 0$$

$$\boxed{\lambda = 1}$$

$$\lambda - 4 = 0$$

$$\lambda = 4$$

So the

eigen values are

$$\lambda = 1, 2, 4$$

Now finding eigenvector

for

$$\lambda = 1$$

Let  $x_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$  be the

eigenvector of  $\lambda = 1$

$$(A - \lambda I) x_1 = 0$$

$$\begin{bmatrix} 2-1 & 1 & 1 \\ 1 & 3-1 & 2 \\ -1 & 1 & 2-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$R_3 \rightarrow R_3 + R_1$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$x + y + z = 0 \quad \text{--- (i)}$$

$$x + 2y + 2z = 0$$

$$0 + 2y + 2z = 0$$

by solving (i)

$$\text{Let } y = k_1 \quad \text{and } z = k_2$$

$$x = -k_1 - k_2$$

$$x_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = k_1 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Hence Eigen Vector

Corresponding  $\lambda = 1$

$$x_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad x_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

End