Q1. a) Differentiate $\dfrac{3x^3 - 5x^2 + 5}{x^2 + 1}$ w.r.t x.

Solution:

$$\frac{3x^3 - 5x^2 + 5}{x^2 + 1}$$

Quotient rule:-

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2}$$

$f(x) = 3x^3 - 5x^2 + 5$

$f'(x) = 9x^2 - 10x$

$g(x) = x^2 + 1$

$g'(x) = 2x$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{(9x^2 - 10x)(x^2 + 1) - (3x^3 - 5x^2 + 5)(2x)}{(x^2 + 1)^2}$$

$$= \frac{9x^2(x^2 + 1) - 10x(x^2 + 1) - 2x(3x^2 - 5x^2 + 5)}{(x^2 + 1)^2}$$

$$= \frac{9x^4 + 9x^2 - \cancel{10x^3} - 10x - 6x^4 + \cancel{10x^3} - 10x}{(x^2 + 1)^2}$$

$$= \frac{3x^4 - 9x^2 - 20x}{(x^2+1)^2}$$ Answer

Q1. b) Differentiate $\frac{(x^2+1)^2}{x^2-1}$ w.r.t x.

Solution:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

$$f(x) = (x^2+1)^2$$
$$f'(x) = 4x(x^2+1)$$
$$g(x) = x^2+1$$
$$g'(x) = 2x.$$

$$\frac{dy}{dx} = \frac{4x(x^2+1)(x^2+1) - (x^2+1)^2(2x)}{(x^2+1)^2}$$

$$= \frac{(4x^3+4x)(x^2+1) - (x^2+1)^2(2x)}{(x^2+1)^2}$$

$$= \frac{4x^3(x^2+1)\,4x(x^2+1) - 2x(x^2+1)^2}{(x^2+1)^2}$$

$$= \frac{4x^5 + 4x^3 + 4x^3 + 4x - 2x\left[(x^2)^2 + 2(x^2)(1) + (1)^2\right]}{(x^2+1)^2}$$

$$= \frac{4x^5 + 8x^3 + 4x - 2x(x^4 + 2x^2 + 1)}{(x^2+1)^2}$$

$$= \frac{4x^5 + 8x^3 + 4x - 2x^5 - 4x^3 - 2x}{(x^2+1)^2}$$

$$= \frac{2x^5 + 4x^3 + 2x}{(x^2+1)^2}$$ Answer.

Q3 a) Find the integration of $\int \frac{1}{\sqrt{x^3}} dx$

Solution:-

$$\int \frac{1}{\sqrt{x^3}} dx = \int \frac{1}{x^{3/2}} dx$$

Applying power rule:

$$\int x^n dx = \frac{x^{n+1}}{n+1}, \quad \text{where } n = \frac{-3}{2}$$

$$= -\frac{2}{\sqrt{x}} + C \quad \text{Answer.}$$

Q3. b) Find integration of $\int \frac{1}{(6x+7)^6} dx$

Solution:

$$\int \frac{1}{(6x+7)^6} dx$$

Substitute $u = 6x+7 \rightarrow \frac{du}{dx} = 6 \rightarrow dx = \frac{1}{6} du$

$$= \frac{1}{6} \int \frac{1}{u^6} du$$

Now solving $\int \frac{1}{u^6} du$

applying power rule:

$$= -\frac{1}{5u^5}$$

putting back the solved integrals:

$$\frac{1}{6} \int \frac{1}{u^6} du = -\frac{1}{30u^5}$$

undo the substitution

$u = 6x + 7$:

$$= -\frac{1}{30(6x+7)^5} + C$$ Answer.