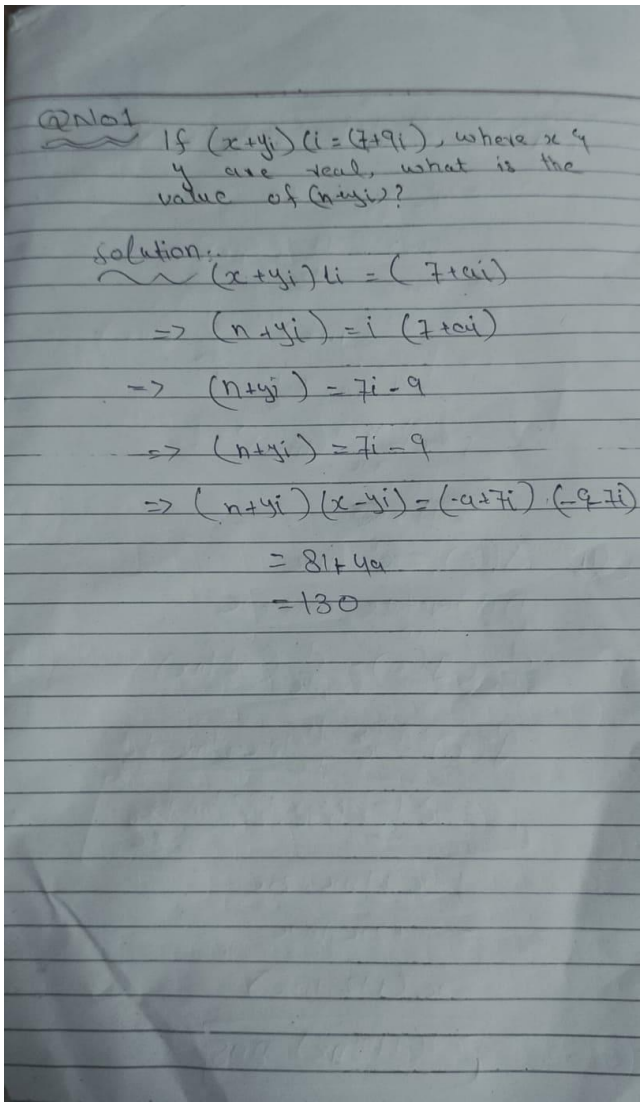
Mid Term Summer 2020 Date: 20 August 2020
Multivariate Calculus
Time Allowed: Four Hours

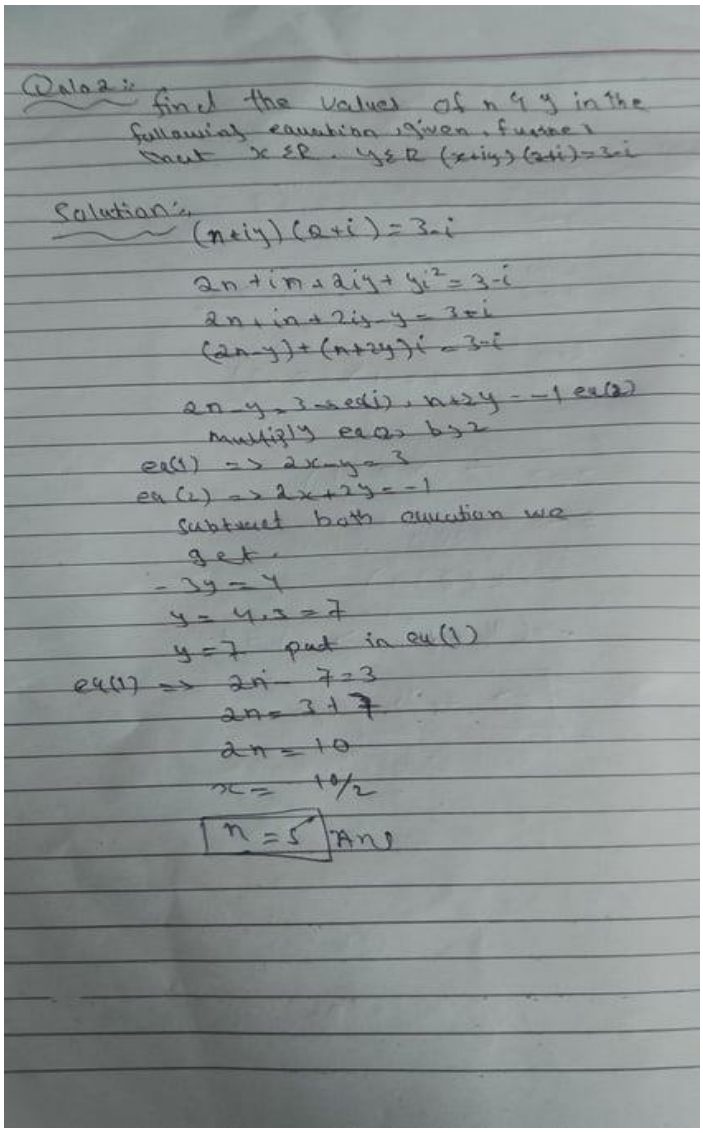
ID 13033

Q.1.If (x + yi) / i = (7 + 9i) , where x and y are real, what is the value of (x + yi)(x - yi)?

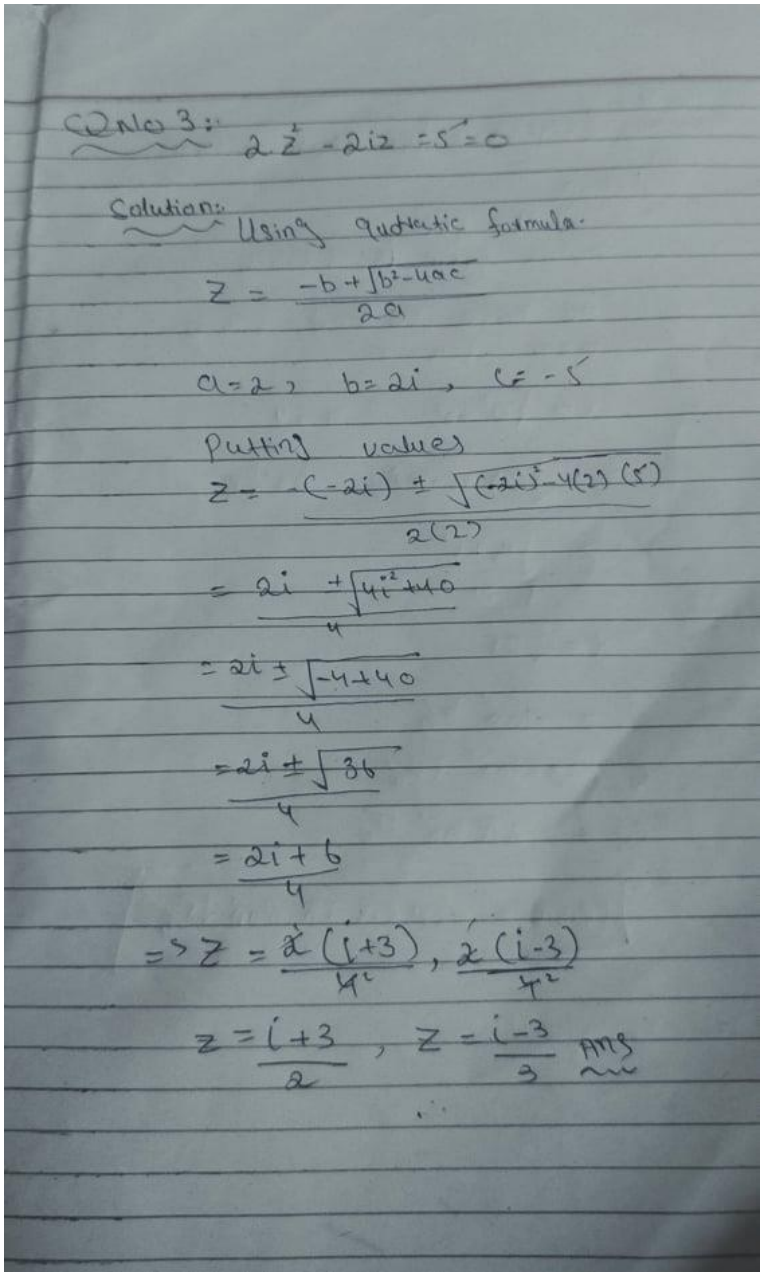


Q.2. Find the values of x and y in the following equation, given further that $x \in R$, $y \in R$.

$(x+iy)(2+i)=3-i$



Q.3. Solve the equation $2z^2 - 2iz - 5 = 0$, $z \in C$.



Q No 3:

$2z^2 - 2iz - 5 = 0$

Solution:

Using quadratic formula.

$$z = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$a = 2$, $b = 2i$, $c = -5$

Putting values

$$z = \frac{-(-2i) \pm \sqrt{(-2i)^2 - 4(2)(5)}}{2(2)}$$

$$= \frac{2i \pm \sqrt{4i^2 + 40}}{4}$$

$$= \frac{2i \pm \sqrt{-4 + 40}}{4}$$

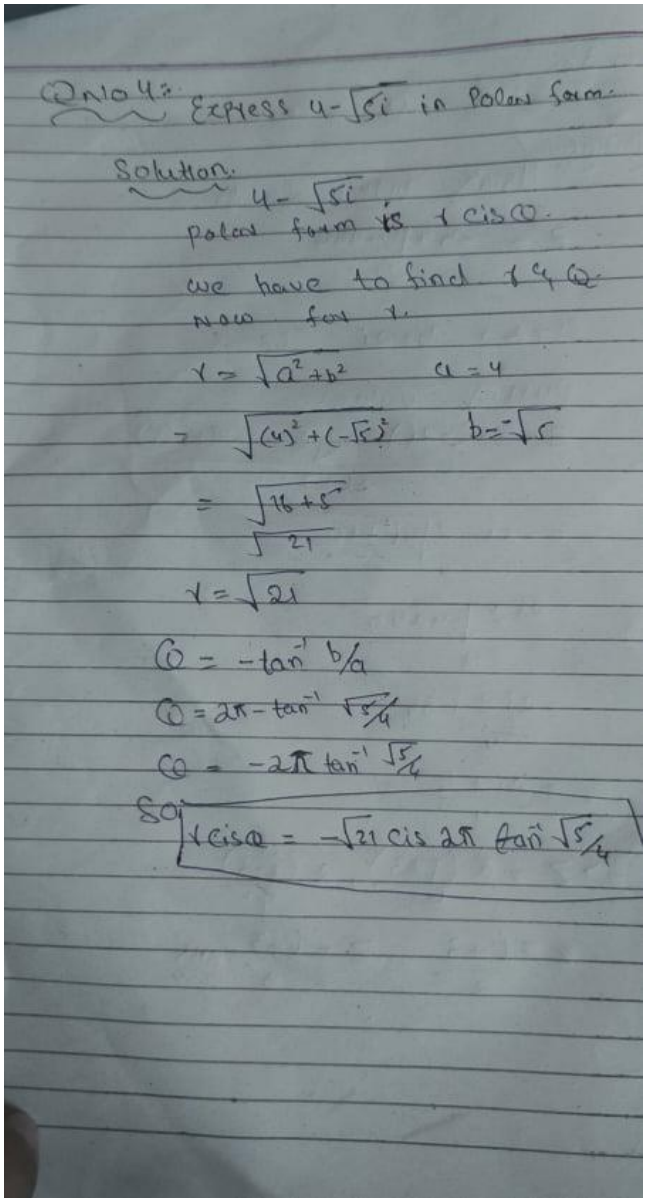
$$= \frac{2i \pm \sqrt{36}}{4}$$

$$= \frac{2i + 6}{4}$$

$$\Rightarrow z = \frac{2(i+3)}{4^2}, \frac{2(i-3)}{4^2}$$

$$z = \frac{i+3}{2}, \quad z = \frac{i-3}{2}$$ Ans

Q.4. Express $4 - \sqrt{5}i$ in polar form.



Q No 4: Express $4-\sqrt{5}i$ in Polar form.

Solution:

$4-\sqrt{5}i$

polar form is $r\,\mathrm{cis}\,\theta$.

we have to find r & θ.

Now for r.

$r = \sqrt{a^2+b^2}$ $\quad a = 4$

$= \sqrt{(4)^2+(-\sqrt{5})^2}$ $\quad b = -\sqrt{5}$

$= \sqrt{16+5}$

$r = \sqrt{21}$

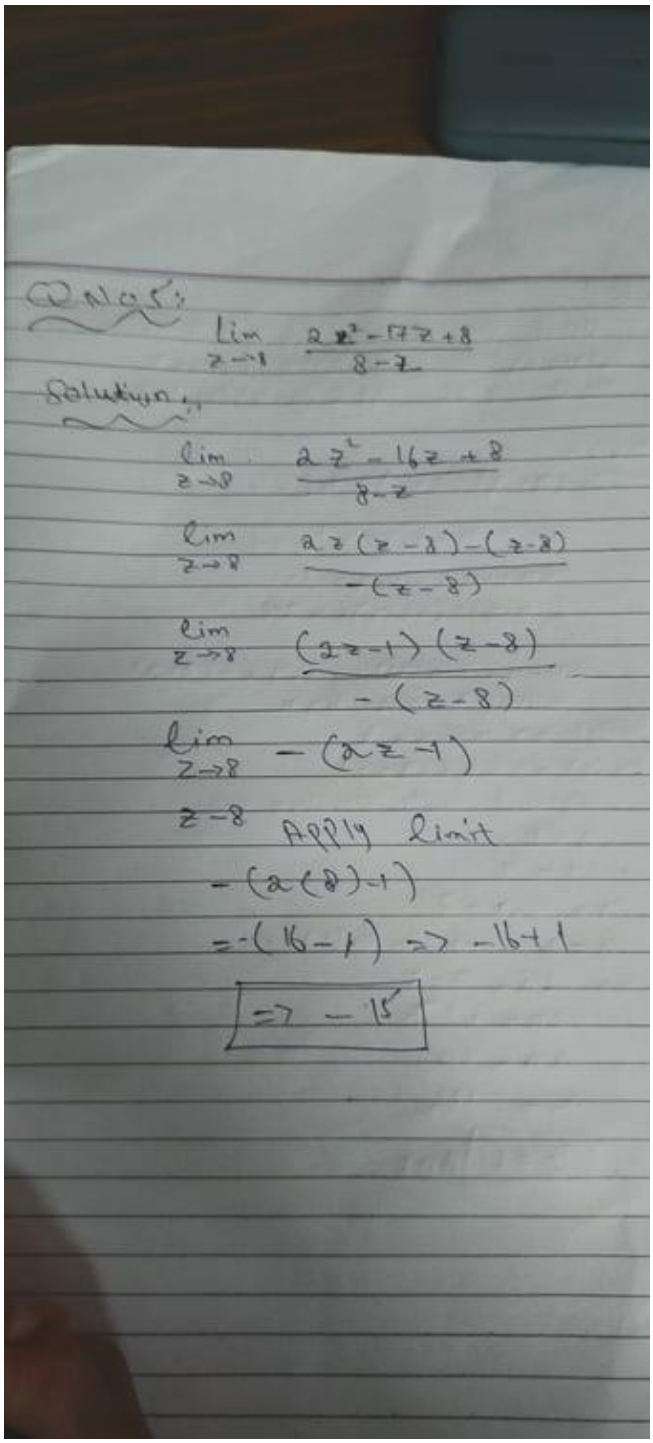
$\theta = -\tan^{-1} b/a$

$\theta = 2\pi - \tan^{-1} \sqrt{5}/4$

$\theta = -2\pi \tan^{-1} \sqrt{5}/4$

So $r\,\mathrm{cis}\,\theta = -\sqrt{21}\,\mathrm{cis}\,2\pi \tan^{-1} \sqrt{5}/4$

Q.5. Find the limit $\lim_{z \to 8} \frac{2z^2 - 17z + 8}{8 - z}$



Q.6.

Differentiate

(i). $f(x)=(\ln x)^4$

(ii). $g(x)=x^2.\ln x$