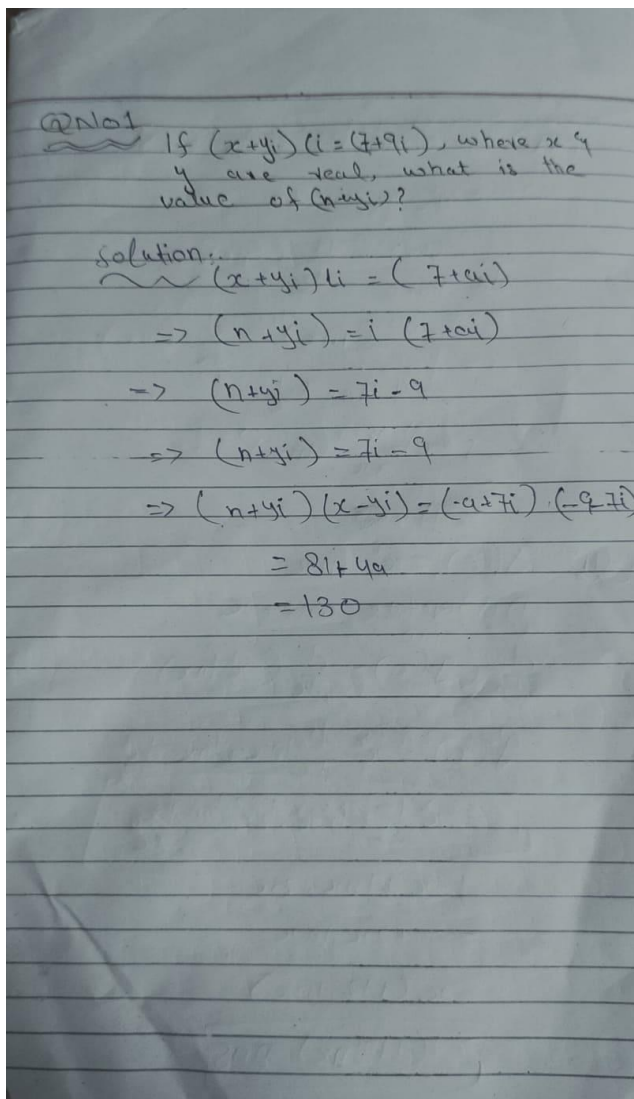


Mid Term Summer 2020
Multivariate Calculus
Time Allowed: Four Hours

Date: 20 August 2020

ID 13033

Q.1. If $(x + yi) / i = (7 + 9i)$, where x and y are real, what is the value of $(x + yi)(x - yi)$?

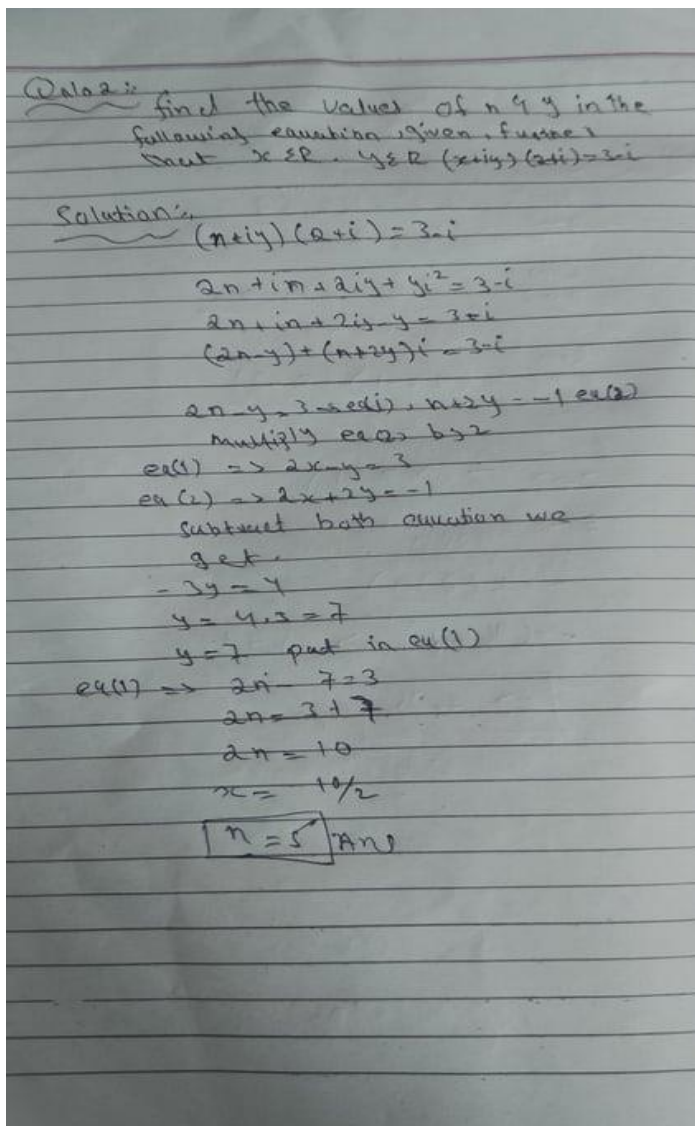


Q.1.1
If $(x + yi) / i = (7 + 9i)$, where x & y are real, what is the value of $(x + yi)$?

Solution:
 $(x + yi) / i = (7 + 9i)$
 $\Rightarrow (x + yi) = i(7 + 9i)$
 $\Rightarrow (x + yi) = 7i - 9$
 $\Rightarrow (x + yi)(x - yi) = (-9 + 7i)(-9 - 7i)$
 $= 81 + 49$
 $= 130$

Q.2. Find the values of x and y in the following equation, given further that $x \in \mathbb{R}$, $y \in \mathbb{R}$.

$$(x+iy)(2+i)=3-i$$



Q.3. Solve the equation $2z^2 - 2iz - 5 = 0$, $z \in \mathbb{C}$.

Q.No 3: $2z^2 - 2iz - 5 = 0$

Solution: Using quadratic formula.

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 2$, $b = 2i$, $c = -5$

Putting values

$$z = \frac{-(-2i) \pm \sqrt{(-2i)^2 - 4(2)(-5)}}{2(2)}$$
$$= \frac{2i \pm \sqrt{4i^2 + 40}}{4}$$
$$= \frac{2i \pm \sqrt{-4 + 40}}{4}$$
$$= \frac{2i \pm \sqrt{36}}{4}$$
$$= \frac{2i \pm 6}{4}$$
$$\Rightarrow z = \frac{2(i+3)}{4}, \frac{2(i-3)}{4}$$
$$z = \frac{i+3}{2}, z = \frac{i-3}{2} \text{ Ans}$$

Q.4. Express $4 - \sqrt{5}i$ in polar form.

Q. No 4: Express $4 - \sqrt{5}i$ in Polar form.

Solution:

$4 - \sqrt{5}i$
polar form is $r \operatorname{cis} \theta$.

we have to find r & θ
now for r .

$$r = \sqrt{a^2 + b^2} \quad a = 4$$
$$= \sqrt{(4)^2 + (-\sqrt{5})^2} \quad b = -\sqrt{5}$$
$$= \frac{\sqrt{16 + 5}}{\sqrt{21}}$$
$$r = \sqrt{21}$$
$$\theta = -\tan^{-1} \frac{b}{a}$$
$$\theta = 2\pi - \tan^{-1} \frac{\sqrt{5}}{4}$$
$$\theta = -2\pi \tan^{-1} \frac{\sqrt{5}}{4}$$

So $r \operatorname{cis} \theta = -\sqrt{21} \operatorname{cis} 2\pi \tan^{-1} \frac{\sqrt{5}}{4}$

Q.5. Find the limit

$$\lim_{z \rightarrow 8} \frac{2z^2 - 17z + 8}{8 - z}$$

Q. No. 5:

$$\lim_{z \rightarrow 8} \frac{2z^2 - 17z + 8}{8 - z}$$

Solukya:

$$\lim_{z \rightarrow 8} \frac{2z^2 - 16z + 8}{8 - z}$$
$$\lim_{z \rightarrow 8} \frac{2z(z - 8) - (z - 8)}{-(z - 8)}$$
$$\lim_{z \rightarrow 8} \frac{(2z - 1)(z - 8)}{-(z - 8)}$$
$$\lim_{z \rightarrow 8} -(2z - 1)$$

$z = 8$ Apply limit

$$-(2(8) - 1)$$
$$= -(16 - 1) \Rightarrow -16 + 1$$
$$\Rightarrow -15$$

Q.6.

Differentiate

(i). $f(x) = (\ln x)^4$

(ii). $g(x) = x^2 \cdot \ln x$

