Course Details

Course Tittle — Linear Algebra

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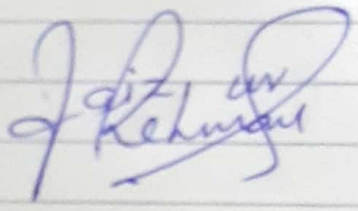
Student sign — 

Q1 Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$ Identify the (3,2) entry of AB.

Sol:→ Identify (3, 2) = ?

$Row_3(A) \cdot Column_2(B)$

$\Rightarrow \begin{bmatrix} 0 & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$

$\Rightarrow (0)(4) + (1)(-1) + (-2)(2)$

$\Rightarrow 0 + (-1) + (-4)$

$\Rightarrow -1 - 4$

$\Rightarrow \boxed{-5}$ Ans

Q1 (b) Part

Label the quadratic polynomial the interpolate the point (1,3) (2,4), (3,7).

Sol$^{n}_{o}$ →

$$a_2 x_1^2 + a_1 x_1 + a_0 = y_1$$
$$a_2 x_2^2 + a_1 x_2 + a_0 = y_2$$
$$a_3 x_3^2 + a_1 x_3 + a_0 = y_3$$

Now $(x_1, y_1) = (1,3)$

$(x_2, y_2) = (2,4)$

$(x_3, y_3) = (3,7)$

Put in Some

$$a_2 + a_1 + a_0 = 3$$
$$4a_2 + 2a_1 + a_0 = 4$$
$$9a_2 + 3a_1 + a_0 = 7$$

$$A_b = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 7 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & -6 & -8 & -20 \end{array}\right] \begin{array}{l} R_2 - 4R_1 \\ R_3 - 9R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & 1 & 4 \end{array}\right] R_3 - 3R_2$$

So

$$a_2 + a_1 + a_0 = 3 \quad - (1)$$

$$-2a_1 - 3a_0 = -8 \quad - (2)$$

$$\boxed{a_0 = 4}$$ Put in (2)

$$-2a_1 - 3a_0 = -8$$

~~$-2a_1 = -12$~~

$$-2a_1 - 3(4) = -8$$

$$-2a_1 - 12 = -8$$

$$a_1 = \frac{4}{2} = -2$$

$$\boxed{a_1 = -2}$$

Put in (1)

$$a_2 + a_1 + a_0 = 3$$

$$a_2 - 2 + 4 = 3$$

$$a_2 + 2 = 3$$

$$a_2 = 3 - 2$$

$$\boxed{a_2 = 1}$$

Q_2 (a) Part :-

If A & B are $n \times n$ matrices where $|A| = 2$ and $|B| = -3$ calculate $|A^{-1} B^t|$.

Sol :->

$$|A^{-1} B^t|$$

$$= |A^{-1}| \, |B^t|$$

$$= \frac{1}{|A|} |B|$$

$$|B^t| = |B|$$

So $$|A^{-1} B^t| = \frac{1}{|A|} |B|$$

$$= \frac{1}{2} \cdot 3$$

$$= \boxed{\frac{3}{2}}$$ Ans

Q2 part b

$$x + y + 2z = 1$$
$$x - 2y + z = -5$$
$$3x + y + z = 3$$

Sol:-

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -6 \\ 0 & -2 & -5 & 0 \end{array}\right] \begin{array}{l} R_2 - R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1/3 & 2 \\ 0 & -2 & -5 & 0 \end{array}\right] \frac{R_2}{-3}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1/3 & 2 \\ 0 & 0 & -13/3 & 4 \end{array}\right] R_3 + 2R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1/3 & 2 \\ 0 & 0 & 1 & -8/13 \end{array}\right] R_3 \times \frac{3}{-13}$$

$$x + y + 2z = 1 \quad - (1)$$
$$y + \frac{1}{3}z = 2 \quad \rightarrow (2)$$
$$\boxed{z = \frac{-8}{13}} \quad \rightarrow (3)$$

Put eqn 2

$$y + \frac{1}{3}x - \frac{8}{13} = 2$$

$$y - \frac{8}{39} = 2$$

$$y = 2 + \frac{8}{39}$$

$$y = \frac{78+8}{39}$$

$$\boxed{y = \frac{86}{39}}$$

Put in eqn 1

$$x + \frac{86}{39} + 2\left(-\frac{8}{13}\right) = 1$$

$$x + \frac{86}{39} - \frac{16}{13} = 1$$

$$x = \frac{38}{39} = 1$$

$$x = 1 - \frac{38}{39}$$

$$\boxed{x = \frac{1}{39}}$$

Q_3 Find A^{-1} where $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$

Sol:-

$$|A| = \begin{vmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{vmatrix}$$

Expand by Row 1

$$= 3\begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} - (-2)\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 3\begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} + 2\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$\Rightarrow 3(-18-0) + 2(-15-2) + 1(0-6)$$

$$\Rightarrow 3(-18) + 2(-17) + 1(-6)$$

$$\Rightarrow -54 - 34 - 6$$

$$\Rightarrow -94$$

$$|A| = -94$$

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} = -18$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -10$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = 2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} = -10$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 28$$

$$\text{adj } A = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}^{t}$$

$$\Rightarrow \begin{vmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{23} \end{vmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{-94} \begin{vmatrix} 18 & 6 & 10 \\ -17 & 10 & 1 \\ 6 & 2 & -28 \end{vmatrix}$$