Iqra National University, Peshawar
Department of Electrical Engineering

IQRA NATIONAL UNIVERSITY

Assignment
Date:20/4/2020

Course Code: MTH 102 **Course Title:** Calculus and analytic geometry
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Module: 3 **Program:** BEE **Total Marks:** 30 **ID NO:** **16194**

Q1.	(a)	. **Identify** $\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$	Marks 5 CLO1 C1
	(b)	Find the first order derivatives of the function $y = \left(x+\frac{1}{x}\right)(x-\frac{1}{x}+1)$	Marks 5 CLO1 C1
Q2	(a)	. A dynamite blast blows up a heavy rock with launch velocity of 160m/sec reaches a height of $s = 160t - 16t^2$ ft after t sec, (i) How high does the rock go (ii) Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down (iii) find the acceleration of the rock at time 5sec	Marks 10 CLO2 C2
Q3	(a)	Does the curve $y = x^4 - 2x^2 + 2$ have nay horizontal tangent if so where?	Marks 10 CLO1 C1

Solotion of Calculas & Analytic geometry

QNo 1 (Part 1)

ANS:-

Given :-

$$\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

Solution :-

$$\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \quad \left(\frac{0}{0}\right)$$

Multiplying & dividing b.s

$\sqrt{2+h} + \sqrt{2}$

$$= \lim_{h \to 0} \frac{\sqrt{2-h} + \sqrt{2}}{h} \times \frac{\sqrt{2+h} + \sqrt{2}}{\sqrt{2+h} + \sqrt{2}}$$

$$= \lim_{h \to 0} \frac{(\sqrt{2-h})^2 - (\sqrt{2})^2}{h(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \to 0} \frac{\cancel{2}+h-\cancel{2}}{h\left(\sqrt{2+h}+\sqrt{2}\right)}$$

$$= \lim_{h \to 0} \frac{1}{\left(\sqrt{2+h}+\sqrt{2}\right)}$$

applying limit

$$= \frac{1}{\sqrt{2+0}+\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}+\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}$$

QNO 1 (Part b)

ANS:

$$y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$$

Solution:

$$y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$$

$$\frac{d}{dx} y = \frac{d}{dx} \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$$

$$= \left(x + \frac{1}{x}\right)\frac{d}{dx}\left(x - \frac{1}{x} + 1\right) + \left(x - \frac{1}{x} + 1\right)\frac{d}{dx}\left(x + \frac{1}{x}\right)$$

$$= \left(x + \frac{1}{x}\right)\left(\frac{d}{dx}x - \frac{d}{dx}\frac{1}{x} + \frac{d}{dx}1\right) + \left(x - \frac{1}{x} + 1\right)$$
$$\left(\frac{d}{dx}x + \frac{d}{dx}\frac{1}{x}\right)$$

$$= \left(x + \frac{1}{x}\right)\left(1 + \frac{1}{x^2}\right) + \left(x - \frac{1}{x} + 1\right)\left(1 - \frac{1}{x^2}\right)$$

$$= x - \cancel{\frac{2}{x}} + \frac{1}{x^3} - \frac{1}{x^2} + 1 + x + \cancel{\frac{2}{x}} + \frac{1}{x^3}$$

$$y' = 2x + \frac{2}{x^3} - \frac{1}{x^2} + 1$$

Q NO # 2

ANS:-

Solutionn

Given $S = 160t - 16t^2$

At any time t the velocity is

$$v = \frac{ds}{dt} = \frac{d}{dt}(160t - 16t^2)$$

$$v = (160 - 32t)$$

(i) at maximum height

$V = 0$

So

$$160 - 32t = 0$$

$$t = \frac{160}{32}$$

$t = 5$ sec

Here

$$S_{max} = S(5) = 160(5) - 16(5)^2$$

$$S_{max} = 400 \, Ft$$

(b) We know that

$$S = 256 \, ft$$

then $160t - 16t^2 = 256$

$$160t - 16t^2 - 256 = 0$$

$$16(t^2 - 10t + 16) = 0$$

$$t^2 - 10t + 16 = 0$$

Factorization

$$t^2 - 8t - 2t + 16 = 0$$

$$t(t-8) - 2(t-8) = 0$$

$$(t-2)(t-8) = 0$$

$$t = 2, \quad t = 8$$

$$t_1 = 8 \, sec, \quad t_2 = 2 \, sec$$

Since

$$v = 160 - 32t$$

at

$$t = 2 \, sec$$

$$V(2) = 160 - 32(2)$$
$$= 96\ m/sec$$

at

$$t = 8$$

$$V(8) = 160 - 32(8)$$
$$= -96\ m/sec$$

(c) Since

$$V = 160 - 32t$$

So

acceleration

$$a = \frac{dV}{dt} = \frac{d}{dt}(160 - 32t)$$

$$= \frac{d}{dt}160 - \frac{d}{dt}32t$$

$$a = -32\ m/sec^2$$

Q NO 3

ANS: Solution:

$$y = x^4 - 2x^2 + 2$$

$$\frac{d}{dx} y = \frac{d}{dx}(x^4 - 2x^2 + 2)$$

$$y' = \frac{d}{dx} x^4 - 2\frac{d}{dx} x^2 + \frac{d}{dx} 2$$

$$y' = 4x^3 - 4x.$$

If the tangent is horizental

$$\frac{d}{dx} \theta = 0$$

$$4x^3 - 4x = 0$$

$$4x(x^2 - 1) = 0$$

$$4x = 0 \quad , \quad x^2 - 1 = 0$$

$$x = 0 \quad , \quad x^2 = 1$$

$$, \quad x = \pm\sqrt{1}$$

$$, \quad x = \pm 1$$

$$x = 0, 1, -1$$

their Coresponding Point is

$y = x^4 - 2x^2 + 2$

for $x = 0$

$y = x^4 - 2x^2 + 2$

$= 0 - 0 + 2$

$y = 2$

for $x = 1$

$y = x^4 - 2x^2 + 1$

$= (1)^4 - 2(1)^2 + 1$

$y = 1$

for $x = -1$

$y = (-1)^4 - 2(-1)^2 + 2$

$= 1 - 2 + 2 = 1$

$y = 1$

Hence

$(0, 2), (1, 1), (-1, 1)$