Name = Mudassir

ID # 7890

Assignment # 02

Department = B.S civil Engr

Q#01:-

The Cauchy Euler Equation

① $x^3 y''' + 2x^2 y' + 2y - 10x + \frac{10}{x}$

Solution:-

$$x^3 \frac{d^3y}{dx} + 2x^2 \frac{dy}{dx} + 2y = 10x + 10x^{-1}$$

$$x^3 D^3 y + 2x^2 D^2 + 2y = 10x + 10x^{-1}$$

$$(x^3 D^3 + 2x^2 D + 2) y = 10x + 10x^{-1} \rightarrow (A)$$

let $x = e^t \Rightarrow t = \ln x$

$$xD = 0$$

$$x^2 D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta$$

$$x^3 D^3 = \Delta(\Delta-1)(\Delta-2)$$

substituting into eq ①

$$(\Delta - 3\Delta^2 + 2\Delta + 2(\Delta^2 - \Delta) + 2)y = 10x + 10x^{-1}$$

$$(\Delta^3 - \Delta^2 + 2)y = 10x + 10x^{-1}$$

$$(m^3 - m^2 + 2)y = 10e^t + \frac{10}{et}$$

using Synthetic division

$$\begin{array}{r|rrrr} & 1 & -1 & 0 & 2 \\ -1 & & -1 & 2 & 2 \\ \hline & 1 & -2 & 2 & \boxed{0} \end{array}$$

$$\Delta^2 - 2\Delta + 2 = 0$$

Now using Quadratic Formula

$$a = 1, \; b = -2, \; c = 2$$

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = \frac{-(-2) \pm \sqrt{-2^2 - 4(1)(-2)}}{2(1)}$$

$$\Delta = \frac{2 \pm \sqrt{4-8}}{2}$$

$$\Delta = \frac{2 \pm \sqrt{-4}}{2}$$

$$\Delta = \frac{2 \pm \sqrt{-1}\sqrt{4}}{2}$$

$$\Delta = \frac{2 + 2i}{2}$$

$$\Delta = \frac{\cancel{2}(1+i)}{\cancel{2}}$$

$$\Delta = 1 + i$$

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Since roots are complex

$y_c = e^{-x}(c_1 \cos t + c_2 \sin t)$

Now Particular Integration

$$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10e^t + \frac{1}{D^3 - D^2 + 2} \cdot {}^{10}/_{e^t}$$

$$= \frac{10e^t}{(1)^3 - (1)^2 + 2} + \frac{10e^{-t}}{(1)^3 - (1)^2 + 2}$$

$$= \frac{\cancel{10}^{5}e^t}{\cancel{2}} + \frac{\cancel{10}^{5}e^{-t}}{\cancel{2}}$$

$$= 5e^t + 5e^t$$

$$y_p = 5e^t + 5e^t$$

General Solution:

$$y = y_c + y_p$$

$$y = e^{-x}(c_1 \cos t + c_2 \sin t) + 5e^t + 5e^{-t}$$

Put $e^t = x$ and $t = \ln x$

$$y = e^{-x}(c_1 \ln x + c_2 \sin \ln x) + 5e^x + 5e^{-x}$$

Q# 2:-

$$\frac{x^3 d^3y}{dx} + 4x^2\frac{d^2y}{dx} - 5\frac{xdy}{dx} - 15y = x^4$$

Sol:-

Let $\frac{d}{dx} = D$

$$x^3D^3y + 4x^2D^2y - 5xDy - 15y = x^4$$

$$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$$

let $x = e^t \Rightarrow t = \ln x$

$$xD = D$$

$$x^2D^2 = \cancel{D0}\ \Delta(\Delta-1) = \Delta^2 - \Delta$$

$$x^3D^3 = \Delta(\Delta-1)(\Delta-2) = \Delta^3 - 3\Delta^2 + 2\Delta$$

Now Substituting:-

$$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$$

$$(\Delta^3 - 3\Delta^2 + 2\Delta + 4(\Delta^2 - \Delta) - 5(\Delta) - 15)y = e^{4t}$$

$$\cancel{(\Delta^3 + \Delta^2 + \Delta)}$$

$$(\Delta^3 + \Delta^2 - 7\Delta - 15)y = e^{4t}$$

synthetic division

5	1	+1	-7	-15
		3	12	15
	1	4	5	$\boxed{0}$

$D^2 + 4D + 5 = 0$

Quadratic formula

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{(-4) \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{(-4) \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm \sqrt{-1}\sqrt{4}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$$= \frac{\cancel{2}(-2 \pm i)}{\cancel{2}}$$

$$y_c = e^{3x}(c_1 \cos t + c_2 \sin t)$$

For $y_p = ?$

$$y_p = \frac{1}{D^3 + D^2 - 7D - 15} \cdot e^{4t}$$

$$= \frac{1}{(4)^3 + (4)^2 - 7(4) - 15} \cdot e^{4t}$$

$$= \frac{1}{80 - 43} \cdot e^{4t}$$

P# 06

$$y_p = \frac{1}{37} e^{4t}$$

Hence

$$y = y_c + y_p$$

$$y = (c_1 \cos t + c_2 \sin t) + \frac{1}{37} e^{4t}$$

Again put $t = \ln x$ and $x = \ln x$

$$y = e^{3x} (c_1 \cos \ln x + c_2 \sin \log x) \frac{1}{37} e^{4t}$$

Pg# 07

Q 5:- $x^2y'' + 2xy' - 6y = 10x^2 \rightarrow (A)$ $y(1)=1,\ y'(1)=-6.$

Sol. A. H. Equation:

$$x^2y'' + 2xy' - 6y = 0 \rightarrow (B)$$

Let, $y = x^m$

Diff: w.r.to x $y' = m x^{m-1}$

Again Diff: w.r.to x $y'' = (m-1)(m)x^{m-2}$

putting values in (B).

$$x^2\left((m^2-m)x^{m-2}\right) + 2x\left(m x^{m-1}\right) - 6x^m = 0$$

$$(m^2-m)x^m + 2m x^m - 6x^m = 0$$

$$(m^2 - m + 2m - 6)x^m = 0$$

$$x^m \neq 0$$

Then $m^2 + m - 6 = 0.$

$$m^2 + 3m - 2m - 6 = 0$$

$$m(m+3) - 2(m+3) = 0$$

$$(m-2)(m+3) = 0$$

$m - 2 = 0$ $m + 3 = 0$

$m = 2$, $m = -3.$

$$y_c = c_1 x^2 + c_2 x^{-3} \longrightarrow (*)$$

For Non-Homogonous Eq: We use variation of parameter method:

$$x^2y'' + 2xy' - 6y = 10x^2$$

$\div$ing by x^2

$$y'' + \frac{2}{x}y' - \frac{6}{x^2} = 10$$

$$f(x) = 10$$

$$y_1 = x^2$$

Now let

$$y_p = u_1 x^2 + u_2 x^{-3} \rightarrow (**)$$

~~By using variation of p~~

$$W = \begin{vmatrix} x^2 & x^{-3} \\ 2x & -3x^{-4} \end{vmatrix} = -3x^{-2} - 2x^{-2}$$

$$= \boxed{-5x^{-2}}$$

$$W_1 = \begin{vmatrix} 10 & x^{-3} \\ 0 & -3x^{-4} \end{vmatrix} = \boxed{-30x^{-4}}$$

$$W_2 = \begin{vmatrix} x^2 & 10 \\ 2x & 0 \end{vmatrix} = \boxed{-20x}.$$

$$u_1' = \frac{W_1}{W} = \frac{\cancel{-}\cancel{30}x^{-4}}{\cancel{-}\cancel{5}x^{-2}} = 6x^{-2}.$$

$$u_2' = \frac{W_2}{W} = \frac{\cancel{-}\cancel{20}x}{\cancel{-}\cancel{5}x^{-2}} = 4x^3$$

$u_1' = 6x^{-2}$	$u_2' = 4x^3$
Taking "$\int$" on b/sides	Taking "$\int$" on b/sides
$\int du_1 = \int 6x^{-2}dx$	$\int du_2 = \int 4x^3 dx$
$u_1 = \frac{6x^{-1}}{-1}$	$u_2 = 4\frac{x^4}{4}$
$\boxed{u_1 = -6x^{-1}}$	$\boxed{u_2 = x^4}$

putting the values in (**)

$$y_p = (-6x^{-1})(x^2) + (x^4)(x^{-3})$$

$$= -6x + x.$$

$$\boxed{y_p = -5x} \rightarrow (***)$$

For [illegible]

$$y = y_c + y_p \rightarrow (xxxx)$$

put (x) & (xxx) in (xxxx)

We got:-

$$y = c_1 x^2 + c_2 x^{-3} - 5x \rightarrow (c)$$

Now we need to find the values of c_1 & c_2.

For this:-

put $y(1) = 1$ in Eq (c).

$$1 = c_1 (1)^2 + c_2 (1)^{-3} - 5(1)$$

$$1 = c_1 + c_2 - 5$$

$$c_1 + c_2 = 6 \rightarrow (a)$$

Diff: Eq (c) w.r.to x

$$y' = 2c_1 x - 3c_2 x^{-2} - 5$$

put $y'(1) = -6$

$$-6 = 2c_1 (1) - 3c_2 (1) - 5$$

$$-6 = 2c_1 - 3c_2 - 5$$

$$2c_1 - 3c_2 = -1$$

$$2c_1 = +3c_2 - 1$$

$$c_1 = \frac{3}{2} c_2 - 1/2 \rightarrow (b)$$

put Eq (b) in Eq (a)

Q4.

$$\left(\frac{3}{2}c_2 - \frac{1}{2}\right) + c_2 = 6$$

xing by '2'

$$3c_2 - 1 + 2c_2 = 12$$

$$5c_2 = 12 + 1$$

$$\boxed{c_2 = 13/5} \rightarrow (d) \text{ put in } (b)$$

$$c_1 = \frac{3}{2}\left(\frac{13}{5}\right) - \frac{1}{2}$$

$$= \frac{39}{10} - \frac{1}{2}$$

$$\boxed{c_1 = \frac{17}{5}} \rightarrow (e)$$

put Eq (d) & (e) in Eq (c)

$$\boxed{y = \frac{17}{5}x^2 + \frac{13}{5}x^{-3} - 5x}$$

The Required General Solution.

Pg# 11

Q4:-

$$x^2y'' + 7xy' + 5y = x^5 \rightarrow (A)$$

$$y(0) = 2, \quad \& \quad y'(1) = 2.$$

Sol:

A.H-Eq:

$$x^2y'' + 7xy' + 5y = 0 \rightarrow (1)$$

Let $y = x^m$

$$y' = mx^{m-1}$$

$$y'' = m(m-1)x^{m-2} \quad \text{put in } (1)$$

$$x^2(m(m-1)x^{m-2}) + 7x(mx^{m-1}) + 5x^m = 0.$$

$$(m^2 - m)x^m + 7mx^m + 5x^m = 0$$

$$(m^2 - m + 7m + 5)x^m = 0.$$

let $x^m \neq 0$

Then

$$m^2 + 6m + 5 = 0$$

$$m^2 + 5m + m + 5 = 0$$

$$m(m+5) + 1(m+5) = 0$$

$$(m+1)(m+5) = 0$$

$$m = -1, \quad m = -5.$$

$$y_c = c_1x^{-1} + c_2x^{-5} \rightarrow (a)$$

(A) $\Rightarrow$

$$x^2y'' + 7xy' + 5y = x^5$$

$$y'' + \frac{7}{x}y' + \frac{5}{x^2}y = x^3 \rightarrow (2)$$

$$y_p = u_1x^{-1} + u_2x^{-5} \rightarrow (*) . \quad f(x) = x^3.$$

$$|W| = \begin{vmatrix} x^{-1} & x^{-5} \\ -x^{-2} & -5x^{-6} \end{vmatrix} = -5x^{-7} + x^{-7} = -4x^{-7} \rightarrow (i)$$

$$|W_1| = \begin{vmatrix} x^3 & x^{-5} \\ 3x^2 & -5x^{-6} \end{vmatrix} = -5x^{-3} - 3x^{-3} \Rightarrow -8x^{-3} \rightarrow (ii)$$

$$|W_2| = \begin{vmatrix} x^{-1} & x^3 \\ -x^{-2} & 3x^2 \end{vmatrix} = 3x + x = 4x \rightarrow (iii)$$

$$u_1' = \frac{w_1}{w}$$

putting values

$$u_1' = \frac{\cancel{7}\,\cancel{8}x^{-3}}{\cancel{7}\,\cancel{4}x^{-7}}$$

$$= 2x^{-3} \cdot x^{+7}$$

$$u_1' = 2x^4$$

Taking "$\int$" on b/s

$$\int u_1' = 2\int x^4 dx$$

$$u_1 = 2 \cdot \frac{x^5}{5}$$

$$\boxed{u_1 = \frac{2}{5} x^5}$$

$$u_2' = \frac{w_2}{w}$$

put

$$u_2' = \frac{\cancel{4}x}{-\cancel{4}x^{-7}}$$

$$u_2' = - x^8 : x^7$$

$$u_2' = - x^8$$

Tak $\int$ on b/s.

$$\int u_2' = - \int x^8 dx$$

$$u_2 = - \frac{x^9}{9}$$

$$\boxed{u_2 = - \frac{x^9}{9}}$$

put in (*)

$$(*) \Rightarrow y_p = \left(\frac{2}{5} x^5\right)(x^{-1}) + \left(-\frac{x^9}{9}\right)(x^{-5})$$

$$y_p = \frac{2}{5} x^4 - \frac{x^9}{9} x^{-5}$$

$$y_p = \frac{2}{5} x^4 - \frac{x^4}{9}$$

$$= x^4 \left(\frac{2}{5} - \frac{1}{9}\right)$$

$$y_p = \left(\frac{18 - 5}{45}\right) x^4$$

$$y_p = \frac{13}{45} x^4 \rightarrow (**)$$

The ~~g~~ General Solution is

$$y = y_c + y_p \rightarrow (***)$$

putti (**) & (a) in (***)

$$y = c_1 x^{-1} + c_2 x^{-5} + \frac{13}{45} x^4 \rightarrow (B)$$

~~put $y(0) = 2$ in (B)~~

~~g~~

$$\boxed{y_1 = c_1 x^{-1} + c_2 x^{-5} + \frac{13}{45} x^4}$$

Q# 5:-

$$(x+1)^2 y'' - 3(x+1)y' + 9 4y = x^2$$

Sol:

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = x^2$$

$$\Rightarrow \left((x+1)^2 \frac{d^2}{dx^2} - 3(x+1)\frac{d}{dx} + 4\right)y = x^2$$

$$\Rightarrow \left[(x+1)^2 D^2 - 3(x+1)D + 4\right]y = x^2 \rightarrow (1)$$

Put $(x+1)D = \Delta \Rightarrow (x+1)^2 D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta$

$x = e^t$ in equ (1)

$$\Rightarrow [\Delta^2 - \Delta - 3\Delta + 4]y = e^{2t}$$

$$\Rightarrow [\Delta^2 - 4\Delta + 4]y = e^{2t}$$

$$\Rightarrow [\Delta^2 - 4\Delta + 4)2 = e^{2t}$$

For y_c we find the roots

$$\Delta^2 - 4\Delta + 4 = 0$$

$$\Delta^2 - 2\Delta - 2\Delta + 4 = 0$$

$$\Delta(\Delta-2) - 2(\Delta-2) = 0$$

$$(\Delta-2)(\Delta-2) = 0$$

$$(\Delta-2) = 0 \quad , \quad (\Delta-2) = 0$$

$$\Delta = 2 \quad , \quad \Delta = 2$$

So roots are equal and ~~reepa~~ repeated

The General Solution are

$$y = (c_1 + c_2 x)^{mx}$$

$$y = (c_1 + c_3 x)^{2x}$$

For $y_p = ?$

$$y_p = \frac{1}{D^2 - 4D + 4}$$

$$(2)^2 - 4(2) + 4$$

$$\Rightarrow 0$$

$$y_p = \frac{2}{2D - 4} e^{2t}$$

If we Put 2

$$2D - 4 \Rightarrow 2(2) - 4 = 0$$

We take again derivative

$$y_p = \frac{\cancel{2}}{\cancel{2}} e^{2t}$$

$$y = (c_1 + c_2 x)^{2t} + e^{2t}$$

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