:- COURSE DETAILS :-

Course Tittle :- Linear Algebra

Module :- 1^{st}

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:- Student Details :-

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Student Sign :-

Q No 1:-

ⓐ:- Express the equation of plane passing through the points $A(2,-2,1)$, $B(-1,0,3)$, $C(5,-3,4)$.

Solution:-

→ The non-parallel Vectors:-

$\overrightarrow{AB} = (-3, 2, 2)$
$\overrightarrow{AC} = (3, -1, 3)$

→ The perpendicular vector is

$$n = \overrightarrow{AB} \times \overrightarrow{AC}$$

$$n = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$$n = i(6+2) - j(-9-6) + k(3-6)$$

$$n = 8i + 15j - 3k$$
$$n = (8, 15, -3)$$

Now $P(x_0, y_0, z_0) = (2, -3, 1)$
$n(a, b, c) = (8, 15, -3)$

So Equation of plane is

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$$8(x-2)+15(y+3)+-3(z-1)=0$$

$$8x-16+15y+45-3z+3=0$$

$$8x+15y-3z+45-16+3=0$$

$$\boxed{8x+15y-3z+32=0}$$ Ans.

part:- (B) Express a pair of planes whose intersection is the given line,
$x = 2-3t$, $y = 3+t$, $z = 2-4t$.

Sol:-

→ $x = 2-3t$

$x-2 = -3t$

$t = \frac{x-2}{-3}$

→ $y = 3+t$

$y - 3 = t$

$t = y-3$

→ $z = 2-4t$

$z-2 = -4t$

$\frac{z-2}{-4} = t$

$t = \frac{z-2}{-4}$

For 1^{st} plane takes 1^{st} and 2^{nd}.

$$\frac{x-2}{-3} = y-3$$

$$x-2 = -3y+9$$

$$\boxed{x+3y-11=0}$$

For 2nd plane take 1^{st} and 3^{rd}.

$$\frac{x-2}{-3} = \frac{z-2}{-4}$$

$$-4x+8 = -3z+6$$

$$-4x+3z+2=0$$

Or $\boxed{4x-3z-2=0}$

Q NO. 2 :-

$$L(x,y) = (x+1,\ y,\ x+y)$$

illustrate that L is linear transformation?

$$L(x,y) = (x+1,\ y,\ x+y)$$

Let $u = (x_1, y_1)$ $\quad v = (x_2, y_2)$

$$U + V = (x_1, y_1) + (x_2, y_2)$$

$$U + V = (x_1 + x_2,\ y_1 + y_2)$$

$$L(U+V) = L(x_1 + x_2,\ y_1 + y_2)$$

$$L(U+V) = (x_1 + x_2 + 1,\ y_1 + y_2,\ x_1 + x_2 + y_1 + y_2) \to (i)$$

Given that $U = (x_1, y_1)$

$$L(U) = L(x_1, y_1) = (x_1 + 1,\ y_1,\ x_1 + y_1)$$

$$L(V) = L(x_2, y_2) = (x_2 + 1,\ y_2,\ x_2 + y_2)$$

$$L(U) + L(V) = (x_1 + x_2 + 2,\ y_1 + y_2,\ x_1 + x_2 + y_1 + y_2) \to (ii)$$

Since equation (i) $\neq$ (ii)

So not L.T :-

QNO. 3:-

Using the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ then interpret to decode the message 77 54 38 71 49 29 68 51 33 76 48 40 86 53 52.

Sol:-

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

To decode the above message we have to break it into five vectors in R^3:

$$\begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix}$$

So solve the equation

$$L(x_1) = \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = Ax_1$$

form since A is non-singular.

$$x_1 = A^{-1}\begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix} = \begin{bmatrix} 16 \\ 8 \\ 15 \end{bmatrix}$$

Similarly

$$x_2 = A^{-1}\begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \\ 7 \end{bmatrix}$$

$$x_3 = A^{-1}\begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix} = \begin{bmatrix} 18 \\ 1 \\ 16 \end{bmatrix}$$

$$x_4 = A^{-1}\begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 12 \end{bmatrix}$$

$$x_5 = A^{-1}\begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 19 \end{bmatrix}$$

Using our correspondence between letters and number, we received the following message

PHOTOGRAPH PLANS

Ans —

→ Q No. 4 :- Find an equation of the plane passing through the point $(-1,3,2)$ and prependicular to the vector $n = (0,1,-3)$.

→ Sol :- Equation of the plane is

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

Given that :-

$$P(x_0, y_0, z_0) = (-1, 3, 2)$$

$$n(a, b, c) = (0, 1, -3).$$

$$0(x-(-1)) + 1(y-3) - 3(z-2) = 0$$

$$0 + y - 3 - 3z + 6 = 0$$

$$y - 3z + 6 - 3 = 0$$

$$\boxed{y - 3z + 3 = 0} \quad \text{Ans.}$$

Q NO. 5 :-

Find an Eigen values and Eigen vectors of matrix

$$A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}.$$

Sol:- we know that $Ax = \lambda x$

$$\begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + x_2 \\ -2x_1 + 4x_2 \end{bmatrix} = \begin{bmatrix} \lambda x_1 \\ \lambda x_2 \end{bmatrix}$$

then

$$x_1 + x_2 = \lambda x_1 \rightarrow (i)$$

$$-2x_1 + 4x_2 = \lambda x_2 \rightarrow (ii)$$

So (i) $x_1 - \lambda x_1 + x_2 = 0$

$$x_1(1-\lambda) + x_2 = 0$$

and (ii)

$$-2x_1 + 4x_2 - \lambda x_2 = 0$$

$$-2x_1 + (4-\lambda)x_2 = 0$$

$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

Characteristic equation.

$$\begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix} = 0$$

$(1-\lambda)(4-\lambda)+2=0$

$4-\lambda-4\lambda+\lambda^2+2=0$

$\lambda^2-5\lambda+6=0$

$\lambda^2-3\lambda-2\lambda+6=0$

$\lambda(\lambda-3)-2(\lambda-3)=0$

$\lambda(\lambda-3)-2(\lambda-3)=0$

$(\lambda-3)(\lambda-2)=0$

$\lambda-3=0$, $\lambda-2=0$

$\lambda_1=3$, $\lambda_2=2$

are eigen Value.

Now find eigen vector of $\lambda_1 = 3$ and put in (i) and (2)

then

$\Rightarrow$ $x_1 + x_2 = 3x_1 \rightarrow$ (i)

$-2x_1 + x_2 = 0$

$2x_1 - x_2 = 0$

$\Rightarrow$ $-2x_1 + 4x_2 = 3x_2 \rightarrow$ (ii)

$-2x_1 + x_2 = 0$

$2x_1 - x_2 = 0$

$x_1 = \frac{1}{2} x_2$

let $x_2 = r$

where $r \neq 0$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} r \\ r \end{bmatrix}$$

eigen vector for $\lambda_2 = 2$ put in (1) and (2)

$x_1 + x_2 = 2x_1 \rightarrow$ (i)

$-2x_1 + 4x_2 = 2x_2 \rightarrow$ (ii)

e.q (i)

$$-x_1 + x_2 = 0$$

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

e.q (ii)

$$-2x_1 + 4x_2 = 2x_2$$

$$-2x_1 + 2x_2 = 0$$

$$x_1 - x_2 = 0$$

$$x_1 = x_2$$

$x_1 = r$ then $x_2 = r$

so $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} r \\ r \end{bmatrix}$