Mid Term Paper ① : Operation Research

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Q1

Orign	1	2	3	4	Supply
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Demand	60	40	30	110	240 / 240

Demand = Supply

Ans Balance Transpotation Problem:

Orign	1	2	3	4	Supply
1	X 20	40 22	X 17	80 4	120 80 40 0
2	10 24	X 37	30 9	30 7	70 40 10
3	50 32	X 37	X 20	X 15	50 0
Demand	60 50	40 0	30 0	110 30 0	

13	13	–	–	
2	2	2	17	
5	5	5	17	
4	(15)	8	3	
4	–	8	3	
8	–	(11)	8	
8	–	–	8	

Total Cost = 40 (22) + 80 (4) + 10 (24) + 30 (9) + 30 (7) + 50 (32)

= 880 + 320 + 240 + 270 + 210 + 1600

total Cost = 3,520

Q2 Min $z = 2x_1 + 3x_2$

Subjected to $= (\frac{1}{2})x_1 + (\frac{1}{4})x_2 \leq 4$

$x_1 + 3x_2 \geq 20$

$x_1 + x_2 = 10$

$x_1, x_2 \geq 0$

Step 1: If any constrain have negative constrain at the right side multiply it by (-1)

$z = -2x_1 - 3x_2$

Step 2: Introduce a slake variable in each $\leq$ constrain

$(\frac{1}{2})x_1 + (\frac{1}{4})x_2 \leq 4$

$= (\frac{1}{2})x_1 + (\frac{1}{4})x_2 + F_1 = 4$

Step 3: Introduce a slack variable & artificial variable in each $\geq$ constrain

$x_1 + 3x_2 \geq 20$

$x_1 + 3x_2 - F_2 + a_1 = 20$

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Step: 4: Introduce an artifical variable in each = to constrain.

$$x_1 + x_2 = 10$$
$$x_1 + x_2 + a_2 = 10$$

Step 5:

For each artifical a_i add $-Ma_i$ to the objective function.

$$3 = -2x_1 - 3x_2$$
$$3 = -2x_1 - 3x_2 - Ma_1 - Ma_2 = 0$$
$$3 + 2x_1 + 3x_1 + Ma_1 + Ma_2 = 0$$
$$2x_1 + 3x_2 + Ma_1 + Ma_2 + 3 = 0$$

Create a table

x_1	x_2	F_1	F_2	a_1	a_2	Z	
1/2	1/4	1	0	0	0	0	4
1	3	0	-1	1	0	0	20
1	1	0	0	0	1	0	10
2	3	0	0	m_1	m_2	1	0

$R_4 + (-m R_3)$

2	3	0	0	m_1	m_2	1	0
$-m$	$-m$	0	0	0	$-m$	0	$-10m$
$2-m$	$3-m$	0	0	m	0	1	$-10m$

x_1	x_2	F_1	F_2	a_1	a_2	Z	
1/2	1/4	1	0	0	0	0	0
1	3	0	-1	1	0	0	20
1	1	0	0	0	1	0	10
$2-m$	$3-m$	0	0	m	0	1	$-10m$

$R_4 - mR_3$

$R_4 + (-mR_2)$

2-m	3-m	0	0	m	0	1	-10m
-m	-3m	0	m	-m	0	0	-20m
2-2m	3-4m	0	m	0	0	1	-30m

	x_1	x_2	F_1	F_2	a_1	a_2	Z	
	1/2	1/4	1	0	0	0	0	4
	1	3	0	-1	1	0	0	20
	1	1	0	0	0	1	0	10
$R_4 - mR_2$	2-2m	3-4m	0	m	0	0	1	-30m

Now we Find Povit column:

x_1	x_2	F_1	F_2	a_1	a_2	Z	
1/2	1/4	1	0	0	0	0	4 = 16
1	3	0	-1	1	0	0	20 = 666
1	1	0	0	0	1	0	10 = 10
2-2m	3-4m	0	m	0	0	1	-30m

Now to Find Povit row:

x_1	x_2	F_1	F_2	a_1	a_2	Z	
1/2	1/4	1	0	0	0	0	4 = 16
1	3	0	-1	1	0	0	20 = 6.66
1	1	0	0	0	1	0	10 = 10
2-2m	3-4m	0	m	0	0	1	-30m

Multiply 4 both side by R_1

x_1	x_2	F_1	F_2	a_1	a_2	Z	
2	1	4	0	0	0	0	
1	3	0	-1	1	0	0	
1	1	0	0	0	1	0	
2-2m	3-4m	0	m	0	0	1	

$R_2 + (-3R_1)$

1	3	0	-1	1	0	0	20
-6	-3	-12	0	0	0	0	-48
-5	0	-12	-1	1	0	0	-28

$R_2 - 3R_1$

x_1	x_2	F_1	F_2	a_1	a_2	Z	
2	1	4	0	0	0	0	16
-5	0	-12	-1	1	0	0	-28
1	1	0	0	0	1	0	10
						1	-30m

$\times \frac{1}{3}$ to R_2

x_1	x_2	F_1	F_2	a_1	a_2	Z	
2	1	4	0	0	0	0	1/16
1/3	1	0	-1/3	1/3	0	0	20/3
1	1	0	0	0	1	0	10
2-2m	3-4m	0	m	0	0	1	-30m

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$R_1 + (-1R_2)$

2	1	4	0	0	0	0	16
$-\frac{1}{3}$	-1	0	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	$-\frac{20}{3}$
$\frac{5}{3}$	0	4	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	$\frac{28}{3}$

	x_1	x_2	F_1	F_2	a_1	a_2	Z	
	$\frac{5}{3}$	0	4	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	$\frac{28}{3}$
	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{20}{3}$
$R_3 + (-1R_2)$	1	1	0	0	0	1	0	10
	2-2m	3-4m	0	m	0	0	1	-30m

$R_3 + (-1R_2)$

1	1	0	0	0	10	10	
$-\frac{1}{3}$	-1	0	$\frac{1}{3}$	$-\frac{1}{3}$	00	$-\frac{20}{3}$	
$-\frac{2}{3}$	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	10	$\frac{10}{3}$	

	x_1	x_2	F_1	F_2	a_1	a_2	Z	
	$\frac{5}{3}$	0	4	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	$\frac{28}{3}$
	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{20}{3}$
	$\frac{2}{3}$	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	1	0	$\frac{10}{3}$
	2-2m	-3-4m	0	m	0	0	-1	-30m

~~$R_4 + (-R)$~~

$R_4 + (-(3-4)R_2)$

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$2-2m \quad 3-4m \quad 0 \quad m \quad 0 \quad 0 \quad 1 \quad -30m$

$-1+4m \quad -3+4m \quad 0 \quad 1-4m \quad -1+4m \quad 0 \quad 0 \quad -10+80m$

$1+2m \quad 0 \quad 0 \quad 1-3m \quad -1+m \quad 0 \quad 1 \quad -10+50m$

	x_1	x_2	F_1	F_2	a_1	a_2	Z	
x_2	$5/3$	0	4	$1/3$	$-1/3$	0	0	$28/3$
x_1	$1/3$	1	0	$-1/3$	$1/3$	0	0	$20/3$
a_2	$3/3$	0	0	$1/3$	$-1/3$	1	0	$10/3$
Z	$1+2m$	0	0	$1-3m$	$-1+4m$	0	1	$-10+50m$

$F_1 = 0$

$x_2 = 28/3$

$x_1 = 20/3$

$a_2 = 10/3$

Ans: $\min Z = -10+50m$

$\max Z = +10+50m$

Q3

Job	A	B	C	D	E
1	6	12	3	11	15
2	4	2	7	1	10
3	8	11	10	7	11
4	16	19	122	23	21
5	9	5	7	6	10

Ans **Phase 1:** Row and column reductions.

Step 1: Subtract the minimum value of each row from the entries of that row.

	A	B	C	D	E	minimum row
1	6	12	[3]	11	15	3
2	4	2	7	[1]	10	1
3	8	11	10	[7]	[11]	7
4	[16]	19	122	23	21	16
5	9	[5]	7	6	10	5

	A	B	C	D	E
1	3	9	0	8	12
2	3	1	6	0	9
3	1	4	3	0	4
4	0	3	106	7	5
5	4	0	2	1	5

Step 2: Subtract the minimum value of each column from the entries of that column:

Job 2:

	A	B	C	D	E
1	3	9	0	8	12
	3	1	6	0	9
	1	4	3	0	4
	0	3	106	7	5
	4	0	2	1	5
Minimum column:	0	0	0	0	4

Job

	A	B	C	D	E
1	3	9	0	8	8
2	3	1	6	0	5
3	1	4	3	0	0
4	0	3	106	7	1
5	4	0	2	1	1

Phase 2: Operation of the problem.

Step1: Draw a minimum number of line to cover all zero of the matrix:

(i) Row scaning:

(ii) Column Scaning:

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Job	A	B	C	D	E
1	3	9	0	8	8
2	3	1	6	0	5
3	1	1	3	0	0
4	0	3	166	7	1
5	4	0	2	1	1

5=5 operator reached:

Job	Operator	Time
1	C	3
2	D	1
3	E	11
4	A	16
5	B	5
		36

Total time cost = 36 chr

Total time Cost = 36 chr Ans

Paper End

Thank You Sir