

Quiz:-

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INU

Section B #

Subject: Differential Equation

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-: Question :-

A yarn merchant sells brand A, B, C of yarn each of which is a blend of Pakistani, Egyptian, & American cotton in the ratio 1:2:1, 2:1:1, 2:0:2 respectively. If one kilogram of A, B, C costs of 40, 50 and 60 rupees respectively, find the cost of a kilogram of cotton of each country.

Solution:

P	E
A	E

1:2:1

P	D
A	E

2:1:1

P	P
A	A

2:0:2

Let x, y, z be the cost/kg of Pakistani, Egyptian, American cotton respectively.

According to given condition:

$$\frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z = 40$$

$$\frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z = 50$$

$$\frac{2}{4}x + \frac{2}{4}z = 60$$

} (1)

(2)

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad u = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad b = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$Au = b$$

$$\left. \begin{array}{l} 1u + 2y + 1z = 160 \\ 1u + 1y + 1z = 200 \\ 1u + 1z = 120 \end{array} \right\} \text{(2)}$$

In Matrix Form:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$A \quad x \quad = \quad b$$

(3)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

$$A_1 = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$$|A| = 1(1 \times 1 - 0 \times 1) - 2(2 \times 1 - 1 \times 1) + 1(2 \times 1 - 1 \times 1)$$

$$|A| = -2.$$

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$$|A_1| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix}$$

$$|A_1| = 160(1 \times 1 - 0 \times 1) - 2(200 \times 1 - 120 \times 1) + 1(200 \times 1 - 120 \times 1)$$

$$|A_1| = -120.$$

$$|A_2| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix}$$

$$|A_2| = 1(200 \times 1 - 120 \times 1) - 160(2 \times 1 - 1 \times 1) + 1(2 \times 1 - 1 \times 200)$$

$$|A_2| = -40.$$

$$|A_3| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{vmatrix}$$

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$$|A_3| = 2(1 \times 120 - 0 \times 200) - 2(2 \times 120 - 1 \times 200) + 160(2 \times 120 - 1 \times 1)$$

$$|A_3| = -120$$

So,

$$|A| = -2$$

$$|A_1| = -120$$

$$|A_2| = -40, \quad |A_3| = -120$$

$$x = \frac{|A_1|}{|A|} = \frac{-120}{-2} = 60$$

$$y = \frac{|A_2|}{|A|} = \frac{-40}{-2} = 20$$

$$z = \frac{|A_3|}{|A|} = \frac{-120}{-2} = 60$$

Result:

$$\text{So, } (x, y, z) = (60, 20, 60)$$