

NAME

M. Bilal

ID

14726

Subject

Probability & Stat

Dept

Bs(CS)

Q1

$$\{(1,1), (1,2), \dots, (6,5), (6,6)\}$$

$$A = \{\text{The sum is 7}\}$$

$$B = \{\text{The sum is even}\}$$

$$C = \{\text{The sum is greater than 8}\}$$

$$D = \{\text{Same outcome}\}$$

Then

$$A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$

$$B = \{(1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$$

$$C = \{(3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6)\}$$

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$$D = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$$

Q2  
Ans:

$$A \cap B = \emptyset$$

$$A \cap C = \emptyset$$

$$A \cap D = \emptyset$$

$$P(A) = \frac{6}{36}, P(B) = \frac{18}{36}, P(C) = \frac{10}{36}$$

$$P(D) = \frac{6}{36}$$

$$P(A \cap B) = 0, P(A \cap C) = 0$$

$$(A \cap D) = 0$$

Hence

$$P(A|B) = 0$$

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = 0$$

$$P(A|D) = 0$$

Q2

Ans: When we are rolling two dice there are 36 different combinations. Counting those up there are 15 possibilities less than 7: (1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1).  
 The probability of getting less than a 7 is

$$\frac{15}{36} = \frac{5}{12}$$

There are 6 possible combinations of getting a 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) which gives a  $\frac{6}{36} = \frac{1}{6}$

This means that 21 possibilities account for getting less than or equal to 7, so there are 15 remaining possibilities of getting more than 7.

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This is the same as  
possibility of getting less  
than 7. So the probability  
must be  $\frac{5}{12}$  as well.

In calculating <sup>12</sup> this we must  
assume that each combination  
is equal likely to all as  
any other and therefore  
the dice are fair.  
or else the calculation  
don't work.

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Q3

Ans.

$$P = \frac{2}{3}$$

$$n = 8$$

$$q = 1 - P$$

$$q = 1 - \frac{2}{3}$$

$$q = \frac{1}{3}$$

i)  $P(x=4)$

$$= \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4$$

$$= \frac{1120}{6561}$$

$$= 0.1707$$

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Q5

Ans

Means and Variance  
of Binomial Random Variable.

The Probability function  
for a binomial random variable  
is

$$b(n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$f(x) = \sum_{x=0}^n \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^n \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^n \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

$$f(n) = \sum_{y=0}^m \frac{(m+1)!}{y!(m-y)!} p^{y+1} (1-p)^{m-y}$$

$$= (m+1)p \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

By Binomial theorem.

$$(a+b)^m = \sum_{y=0}^m \frac{m!}{y!(m-y)!} a^y b^{m-y}$$

Set  $a = p$  &  $b = 1-p$

$$\sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$\frac{(a+b)^m}{(p+1-p)^m}$$

$$= 1$$

So that

$$f(n) = np$$

$$y = n-1 \quad \& \quad m = n-1$$

$$f(n) = \sum_{h=0}^n n(n-1) \binom{n-1}{h} p^h (1-p)^{n-1-h}$$

$$\sum_{h=0}^n n(n-1) \frac{(n-1)!}{h!(n-1-h)!} p^h (1-p)^{n-1-h}$$



So the variance of  $n$  is  
$$E(n^2) - f(n) = f(n(n-1) + f(n))$$

$$f(n) = (n(n-1)p^2 + np) - (np) \\ = np(1-p).$$

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(Law of total Probability)

Hence A & B are independent.

Q6

Bi-nomial Distribution:

A Bi-nomial distribution can be thought of as simply the Probability of a Success or failure outcome in an experiment or survey that is repeated multiple times.

$$P(n) f(n) = \binom{n}{x} p^x q^{n-x}$$

Bi-nomial Frequency Distribution:

If the Bi-nomial probability distribution is multiplied by  $n$ ; number of experiment or sets the resulting

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A survey distribute is  
is known as the bi-nomial  
frequency distribution.

$$N \binom{n}{x} p^x q^{n-x}$$

Q7

Coefficient of Variation.

Data Set A:-

$$CV = \frac{\sigma}{\mu} \times 100$$

$$CV = \frac{3}{45} \times 100$$

$$CV = 6.7$$

Data Set B:-

$$CV = \frac{11}{60} \times 100$$

$$CV = 18.3$$

A survey distribute is known as the bi-modal frequency distribution.

$$N \binom{n}{r} p^r q^{n-r}$$

Q7

Coefficient of Variation.

Data Set A:-

$$CV = \frac{\sigma}{\mu} \times 100$$

$$CV = \frac{3}{45} \times 100$$

$$CV = 6.7$$

Data Set B:-

$$CV = \frac{11}{60} \times 100$$

$$CV = 18.3$$

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Data Set C:

$$CV = \frac{5}{50} \times 100$$

$$CV = 10$$

Data Set D:-

$$CV = \frac{15}{25} \times 100$$

$$CV = 60$$

$$\text{ii) } P(X \geq 4)$$

iii)

$$1 - P(X < 4)$$

$$= 1 - \sum_{x=0}^3 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$$

$$1 - \left[ \binom{8}{0} \left(\frac{1}{3}\right)^8 + 8 \binom{8}{1} \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)^7 + 28 \binom{8}{2} \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^6 + 56 \binom{8}{3} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 \right]$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561}$$

$$= \frac{5984}{6561}$$

$$= 0.9121$$

$$\text{iii) } P(3 \leq x \leq 6)$$

$$\sum_{x=3}^6 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 + \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 + \binom{8}{5}$$

$$\left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^3 + \binom{8}{6} \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^2$$

$$= \frac{8}{(3)^8} [56 + 140 + 224 + 224]$$

$$= \frac{8 \times 644}{6561}$$

$$= \frac{5152}{6561}$$

$$= 0.7852$$