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QUESTION #1

FOURIER SERIES:

$$f(t) = 1+t, \; -\pi \leq t \leq \pi$$

Here we use formula

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \quad \text{①}$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt$$

$$a_0 = \frac{1}{2\pi} \left| t + \frac{t}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(\frac{-\pi^2}{2} \right) \right)$$

$$a_0 = \frac{1}{2\pi} \left(2\pi + \pi^2 \right)$$

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$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)(\cos nt)\,dt$$

$$a_n = \frac{1}{\pi}\left((1+t)\,\frac{\sin nt}{n}\Big|_{-\pi}^{\pi} - \int\left(\frac{\sin nt}{n}\frac{d}{dt}(1+t)\right)\right)$$

$$a_n = \frac{-1}{\cancel{n^2}\,\pi}\left((1+t)\,\frac{\sin \cancel{n}t}{\cancel{n}}^{0} - \frac{\cos nt}{n^2}\Big|_{-\pi}^{\pi}\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(-1-(-1)\right)$$

$$a_n = 0$$

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$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)\sin nt\,dt$$

$$b_n = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi}\sin nt - \int\left(\int \sin nt \cdot \frac{d}{dt}(1+t)\,dt\right)\right)$$

$$b_n = \frac{1}{\pi}\left(\left.(1+t)\frac{(-\cos nt)}{n}\right|_{-\pi}^{\pi} - \int\left(\frac{-\cos nt}{n}(1)\right)\right)$$

$$b_n = \frac{1}{\pi}\left(\left.\frac{-(1+t)(\cos nt)}{n}\right|_{-\pi}^{\pi} + \left(\left.\frac{\sin \cancel{n}\overset{0}{\pi}}{\cancel{n}^2}\right|_{-\pi}^{\pi}\right)\right)$$

$$\cancel{b_n = \frac{-1}{n\pi}\left((1+\pi)(\cos nt)\Big|_{-\pi}^{\pi}\right)}$$

$$b_n = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - ((1+(\pi))(\cos n\pi)\right)$$

$$b_n = \frac{-1}{n\pi}\left(\cancel{\cos n\pi} + \pi\cos n\pi - \cancel{\cos n\pi} + \pi\cos n\pi\right)$$

$$b_n = \frac{-1}{n\cancel{\pi}}(2\cancel{\pi}(\cos n\pi)$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

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$$b_n = \frac{2}{n}(-1)^{n+1}$$

so equation becomes

x ——— x ——— x ——— x ——— x ——— x

$$f(x) = \frac{1}{2\pi}(2\pi + \pi^2) + 0 + \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \sin t$$

x ——— x ——— x ——— x ——— x ———

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QUESTION #2.

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen values =?

SOLUTION:-

* we howe;

$$(A - \lambda I)x = 0$$

* we have; the characteristics equation is given by.

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

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★ $\lambda^3 - \left|\begin{matrix}\text{sum of} \\ \text{diogonal} \\ \text{element}\end{matrix}\right| \lambda^2 + \left|\begin{matrix}\text{Sum of} \\ \text{diogonal} \\ \text{minor}\end{matrix}\right| \lambda - |A| = 0$ — (B)

Sum of diogonal elements $= 1+1+2 = 4$

sum of diogonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= -6 + 2 + 1$

$= -3$

By putting values in eq B.

$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0$ — (C)

$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$

$= 1(2-8) - 0 + 1(6-0)$

$= -6 + 6$

$= 0$

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By putting value in eq(c)

$\lambda^3 - 4\lambda^2 - 3\lambda = 0$

$\lambda^3 - 4\lambda^2 - 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda - 3) = 0$

$\lambda = 0$

$\lambda^2 - 4\lambda - 3 = 0$

Using Quadratic equation.

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$
$b = -4$
$c = -3$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4 + \sqrt{28}}{2}, \quad \lambda = \frac{4 - \sqrt{28}}{2}$$

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We have eigen values;

$$\lambda = \left(0,\ \frac{4+\sqrt{28}}{2},\ \frac{4-\sqrt{28}}{2}\right)$$

ANSWER.

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QUESTION # 3

solve the following system of linear equation:

$$5x + 0 - 4z + 2m = 3$$
$$x - y + 2z + m = 1$$
$$4x + y + 2z + 0 = 1$$
$$x + y + z + m = 0$$

Solution:

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \quad R_4 R_2$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1+4/5 & 1 \\ 0 & -1 & +6/5 & +4/5 & 3/5 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \quad 1/5 \times R_3$$

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$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right] \quad 5 \times R_3 \;\&\; 5 \times R_4$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 5R_3 \;\&\; 5R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 1/5 \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_2 \times 5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3 - R_2$$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$R_3 \leftrightarrow R_4$

$1/7 \times R_3$

$1/3 \times R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$C_2 \times -5$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$5/4 \times R_1$

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$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$(x, y, z, m) = (3/4, 31/21, -11/21, 1/3)$$

$x = 3/4$

$y = 31/21$

$z = -11/21$

$m = 1/3$

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QUESTION # 4

Verify that

$$u(x,t) = \sin(x + 2t)$$

is a solution of one dimensional equation.

Solution:

Given that

$$u(x,t) = \sin(x + 2t)$$

diff wrt x partially,

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \sin(x + 2t)$$

$$\frac{\partial u}{\partial x} = \cos(x + 2t) \frac{\partial}{\partial x}(x + 2t)$$

$$\frac{\partial u}{\partial x} = \cos(x + 2t)(1 + 0)$$

$$\partial u/\partial x = \cos(x + 2t)$$

$$\partial^2 u/\partial x^2 = \partial/\partial x \cos(x + 2t)$$

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$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)\frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)(1+0)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)$$

and

$$u(x,t) = \sin(x+2t)$$

diff w.r.t t.

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t)(0+2)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial t^2} = (2)-\sin(x+2t)(0+2)$$

$$\frac{\partial^2 u}{\partial t^2} = -4\sin(x+2t)$$

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as we know that one dimensiol equation is.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$-4\sin(x+2t) = c^2\left[-\sin(x+2t)\right]$$

$$-4\sin(x+2t) = -c^2 \sin(x+2t)$$

$$-4\sin(x+2t) + c^2 \sin(x+2t) = 0$$

for the orbitary constant

$$c = \pm 2$$

$$-4\sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

than it will be verified for the orbitory constant.

$$c = 2$$

ANSWER.