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QNO# 01

Find PQ where P is The point in Three-dimensional space with coordinats (4,1,3) & the Point Q with coordinats (1,2,4).

Find the distance b/w P & Q

Further, Find the position vector of the Point dividing PQ in the ratio 1:3

Sol

Coordinate of $P = (4,1,3)$

$$OP = 4i + 1j + 3K$$

OR

$$OQ = \vec{OQ} - \vec{OP}$$

$$= (i + 2j + 4k) - (4i + 1j + 3k)$$

$$= -3i + 1j + 1k \longrightarrow (i)$$

Now the distance b/w P & Q

$$= |PQ|$$

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$= \sqrt{11} \longrightarrow (2)$

Let M be the point which divided PQ in ratio 1:3, then by ratio Theorm Position vector of $M = \overrightarrow{OM}$

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1+3}$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13i + 5j + 13k}{4} \longrightarrow (3)$$

Hence eq (1), (2) and (3) are the required Sol

QNO # 02

* Evaluate

$$\int \frac{4x^3+10x+4}{2x^2+x}$$

Sol:

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x^2+x & \cancel{4x^3}+10x+4 \\ & \pm\cancel{4x^3} \quad \pm 2x^2 \\ \hline & -2x^2+10x+4 \\ & \mp 2x^2 \pm x \\ \hline & 11x+4 \end{array}$$

So,

$$2x-1+\frac{11x+4}{2x^2+x} = \frac{4x^3+10x+4}{2x^2+x}$$

$$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+x} = \int 2x-1+\int \frac{11x+4}{2x^2+x} \longrightarrow (1)$$

$$\Rightarrow 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$\Rightarrow \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \longrightarrow 2$$

Now Find

$$= \int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$= \frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \longrightarrow A$$

$$= \frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$\Rightarrow 11x+4 = A(2x+1) + Bx \longrightarrow 3$$

Put $x=0$ in 3

$$\boxed{4 = A}$$

Now put $x = -1/2$ in 3

$$= 11\left(-\tfrac{1}{2}\right) + 4 = B\left(-\tfrac{1}{2}\right)$$

$$= -\tfrac{11}{\cancel{2}} + 4 = -\tfrac{B}{\cancel{2}}$$

$$\Rightarrow -3 = -B \Rightarrow \boxed{B = 3}$$

Now putting the values of A and B in eq (A).

$$= \frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking Integral on both sides.

$$= \int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4\,dx}{x} + \int \frac{3}{2x+1}\,dx.$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + \tfrac{3}{2}\ln|2x+1|$$

Putting these values in eq (2)

$$= x^2 - x + 4\ln|x| + \tfrac{3}{2}\ln(2x+1)$$

Now put these values in eq (1).

$$= \int \frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x| + \frac{3}{2}$$

$$\ln(2x+1) + C$$

QNO $\boxed{03}$ (A) Part

(a) $\int_0^2 x^2 e^x \, dx.$

Sol Now Frist Find Integration

$$= \int_0^2 x^2 e^x \, dx.$$

$$= x^2 \int e^x dx - \int \left(e^x dx \frac{d}{dx} x^2 \right) dx.$$

$$= x^2 e^x - \int e^x (2x) \, dx.$$

$$= x^2 e^x - 2 \int x e^x \, dx.$$

$$= x^2 e^x - 2 \left[x \int e^x - \int e^x \, dx \right]$$

$$= x^2 e^x - 2x e^x + 2e^x$$

⇒ Now put limits.

$$= \left| x^2 - 2x e^x + 2e^x \right|_0^2$$

$$= \left(2^2 e^2 - 2(2) e^2 + 2e^2 - (0 - 0 + 2e^0) \right)$$

$$= \left(\cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2 \right)$$

$$\Rightarrow \boxed{2e^2 - 2} \quad \text{Ans}$$

QNO 03 (B) part

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx.$$

Sol Frist Find Integration

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx = ? \quad \text{———} ①$$

let $y = \sqrt{x}.$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}.$$

$2dy = \frac{1}{\sqrt{x}}\, dy$ Put eq ①

$$\int \sin(y)\,(2dy) = 2\int \sin(y)\, dy$$

$$= 2(-\cos y)$$

$= -2\cos y$ Put $y = \sqrt{x}.$

$$= -2\cos\sqrt{x}.$$

Put limit.

$$= -2\left|\cos\sqrt{x}\right|_1^2 = -2(\cos\sqrt{x} - \cos 1)$$

$$= \boxed{-2\cos\sqrt{2} + 2\cos(1)}$$

Q NO (04).

Verify That.

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfies that three dimensional place equation

Sol The laplace eq in 3 dimensional.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad \text{— (A)}$$

So,

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}.$$

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}.$$

$$\frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$\frac{\partial u^2}{\partial x^2} = -x\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{\partial u^2}{\partial x^2} = -\left[x\left(\frac{-3}{\cancel{2}}\right)\left(x^2+y^2+z^2\right)^{-5/2}(\cancel{2}x) + \left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$\frac{\partial u^2}{\partial x^2} = 3x^2\left(x^2+y^2+z^2\right)^{-5/2} + \left(x^2+y^2+z^2\right)^{-3/2} \quad \text{— } 1$$

NOW

$$\frac{\partial u}{\partial y} = \frac{-1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}(\cancel{2}y)$$

$$\frac{\partial u}{\partial y} = -y\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{\partial u^2}{\partial y^2} = -\cancel{y}\left[y\left(\frac{-3}{\cancel{2}}\right)\left(x^2+y^2+z^2\right)^{-5/2}(2y) + \left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$\frac{\partial u^2}{\partial y^2} = 3y^2\left(x^2+y^2+z^2\right)^{-5/2} + \left(x^2+y^2+z^2\right)^{-3/2} \quad \text{— } 2$$

$$\frac{\partial u}{\partial z} = -\frac{1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}(\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = -z\left(x^2+y^2+z^2\right)^{-3/2}.$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2} \rightarrow 3$$

Putting eq 1. 2 and 3 in eq A.

$$= 3x^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2} + 3y^2\left(x^2+y^2+z^2\right)^{-5/2}$$
$$- \left(x^2+y^2+z^2\right)^{-3/2} + 3z^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2}$$

$$= \left(x^2+y^2+z^2\right)^{-5/2}\left[3x^2 - \left(x^2+y^2+z^2\right) + 3y - \left(x^2+y^2+z^2\right) + 3z^2 - \left(x^2+y^2+z^2\right)\right]$$

$$= \left(x^2+y^2+z^2\right)^{-5/2}\left[\cancel{3x^2} - \cancel{x^2} - \cancel{y^2} - \cancel{z^2} + 3y^2 - \cancel{x^2} - \cancel{y^2} - z^2 + 3z - \cancel{x^2} - \cancel{y^2} - z^2\right]$$

$$= \left(x^2+y^2+z\right)^{-5/2}(0) = 0$$

So The given $u(x, y, z)$ is Solution of Laplace equation.