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Section " B "

Fourth Semester

Subject : Differential Equation

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Q No #01. Solve the following objective type Questions.

(i) The order of Matrix A is $m \times p$ and the order of B is $p \times n$. Then the Order of matrix AB is?

Solution:→ The order of A matrix is equal to the Number of its Rows Multiply by Number of its Column.

So $A = m \times p$ where "m" number of Rows and "p" number of Column.

Similerly $B = p \times n$ where "p" Numbers of Row and "n" number of column.

Now According to the Rule Number of Column in A is equal to the Numbers of Rows in B. Then we can Multiply A and B. that as.

$$\boxed{AB = m \times n}$$

——— × × ———

(ii) The Number of Non-Zero rows in an Echelon form?

Sol :→ The Number of Non-Zero rows in a Echelon form is of a Matrix is called "RANK".

(iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular Matrix than $a = ?$

Solution :→ For Singular Matrix $|B| = 0$

so $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$

& $|B| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix} = 1 \times a - 4 \times 2 = 0$

$$a - 8 = 0$$

$$\boxed{a = 8}$$

(iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ Then $|A| = ?$

Sol :→ $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$= (2i)(-i) - (i)(i)$$

$$= -2i^2 - i^2$$

we know that $i^2 = -1$

$$= -2(-1) - (-1)$$

$$= 2 + 1$$

$$\boxed{|A| = 3}$$

(V) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is = ?

Sol:→ If each element of a Principle diagnal of a matrix is some non-zero Scaler ~~at~~ and all other elements are zero that it is a scalar Matrix so,

$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is a scalar Matrix.

(vi) Solution of $\frac{dy}{dx} + 2xy = y$ is ?

Solution :→ $\frac{dy}{dx} + 2xy = y$

$\Rightarrow \frac{dy}{dx} = y - 2xy$

$\Rightarrow \frac{dy}{dx} = y(1-2x)$

$\Rightarrow \frac{dy}{y} = (1-2x)\,dx$

$\Rightarrow \int \frac{dy}{y} = \int 1\,dx - 2\int x\,dx$

$\Rightarrow \ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$

$\Rightarrow \ln y = x - x^2 + c$

(vii) The order and degree of differential equation.

$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is = ?

Sol :→ The order of differential equation is the order of highest order of derivatives and degree of highest order derivatives.

Order = 1 , Degree = 3

(viii) The order and degree of differential $\frac{d^2y}{dx^2} - 4xy = \sin \frac{d^2y}{dx^2}$ is?

Solution: $\rightarrow$ So Order $= 2$

Degree is Not defined.

(ix) The differential equation

$$2\frac{dy}{dx} + x^2 y = 2x + 3 \quad , y(0) = 5$$

is $=$?

Sol: $\rightarrow$

$$2dy + x^2 y = (2x+3)\,dx$$

$$2dy = (2x + 3 - x^2 y)\,dx$$

$$2\int dy = 2\int x\,dx + 3\int dx - y\int x^2\,dx$$

$$2y = \frac{\cancel{2}x^2}{\cancel{2}} + 3x - \frac{yx^3}{3} + c$$

$$2y = x^2 + 3x - \frac{yx^3}{3} + c$$

Both side divided By 2

we get $y = \frac{x^2}{2} + \frac{3x}{2} - \frac{yx^3}{6} + c$

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{yx^3}{6} + c \quad \text{— (A)}$$

initial Condition

$y(0) = 5$ so $x = 0, y = 5$

Put in equation (A)

$$5 = \frac{(0)^2}{2} + \frac{3(0)}{2} - \frac{y(0)^3}{6} + c$$

$$5 = 0 + 0 - 0 + c$$

$$\boxed{c = 5}$$

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{yx^3}{6} + c$$

(X) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Sol:→ $\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = |A|$

Expand By R_1

$$|A| = +1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$= +1(bc^2 - b^2c) - a(c^2 - b^2 + a^2(c - b)$$

$$\boxed{|A| \Rightarrow bc^2 - b^2c - ac^2 + ab^2 + ac^2 - a^2b}$$

Q No # 02

part (i) Express The determinant $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$ as The product of factors which are linear in a, b, c.

Solution:→

let $A = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

Then

$$|A| = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$$|A| = a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$|A| = a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$|A| = ab^2c^3 - ab^3c^2 - ba^2c^3 + ba^3c^2 + ca^2b^3 - ca^3b^2$$

$$|A| = abc(bc^2 - b^2c - ac^2 + ab^2 - a^2b) \rightarrow \text{Ans}$$

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Q No#02

part (ii)

Find Eigen Value $\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

Solution:→

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Characteristic equation $\Rightarrow |A-\lambda I|=0$ — (A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now we take The Determinante

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand By R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} -(-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$- \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \quad \text{——— (B)}$$

Again Expand By R_1

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow (3-\lambda)\left[(3-\lambda)(2-\lambda)-(-1)(-1)\right] + 1\left[(-1)(2-\lambda) - (-1)(-1)\right] - 1\left[(-1)(-1) - (-1)(3-\lambda)\right]$$

$$\Rightarrow (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (+1+3-\lambda)$$

$$\Rightarrow (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-3+\lambda) - (4-\lambda)$$

$$\Rightarrow 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= \boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \rightarrow ⓐ$$

$$\Rightarrow \quad +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand By C_1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & 1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1\,(6 - 3\lambda - 2\lambda + \lambda^2 - 1) + 1\,(-2 + \lambda - 1)$$

$$\Rightarrow -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$\Rightarrow \boxed{-\lambda^2 + 6\lambda - 8} \longrightarrow b$$

$$\Rightarrow \quad -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by Column 1

$$\Rightarrow -\left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -\left[-(-2 + \lambda - 1) + 1\,(6 - 3\lambda - 2\lambda + \lambda^2 - 1)\right]$$

$$\Rightarrow -(3 - \lambda + \lambda^2 - 5\lambda + 5)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$\Rightarrow \boxed{-\lambda^2+6\lambda-8} \text{ — } (c)$$

Put eq (a), (b), and (c) in (B)

$$\Rightarrow (2-\lambda)[-\lambda+8\lambda^2-18\lambda+8]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$$

$$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2$$
$$+6\lambda-8-\lambda^2+16\lambda-8$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda$$
$$+6\lambda+6\lambda+16-16$$

$$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$$

By Synthatic Dirision

we get:

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$$

$$(\lambda=0)$$

$$\lambda-2=0 \quad \boxed{\lambda=2}$$

$$\lambda^2-8\lambda+16=0$$

$$\lambda^2 - 8\lambda + 16 = 0$$

By factorization Method..

$$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$$

$$\lambda(\lambda - 4) - 4(\lambda - 4)$$

$$(\lambda - 4)(\lambda - 4)$$

$$\lambda = 4, \lambda = 4$$

So $\boxed{\lambda_1 = 0, \lambda_2 = 2, \lambda_3 = 4, \lambda_4 = 4}$

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QNO#03 The rate of change in the form of differential equation is given by $(x^2 + 3y^2)dx - 2xy\,dy = 0$ Find Genral Solution at $x = 2$ & $y = 6$

Solution:→

So

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying Both Side By "2"

$$\Rightarrow 2v + 2x\frac{dv}{dx} = \left[\frac{1}{v} + 3v\right]$$

$$\Rightarrow 2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$\Rightarrow$ By taking lcm.

$$\Rightarrow 2x\frac{dv}{dx} = \cancel{v} * \frac{1}{\cancel{v}} + \frac{3v}{1} * v - \frac{2v}{1} * v$$

$$\Rightarrow 2x\frac{dv}{dx} = \frac{1 + 3v^2 - 2v^2}{v}$$

$$\Rightarrow 2x\frac{dv}{dx} = \frac{1 + v^2}{v}$$

$$\Rightarrow 2x\,dv = \left(\frac{1+v^2}{v}\right)dx$$

$$\Rightarrow \frac{v}{1+v^2}\,dv = \frac{dx}{2x}$$

$$\Rightarrow \int \frac{2v}{1+v^2}\,dv = \int \frac{dx}{x}$$

$$\Rightarrow \ln|1+v^2| = \ln x + \ln c$$

$$\cancel{\ln}|1+v^2| = \cancel{\ln}\, xc$$

$$1+v^2 = xc$$

Put $v = \frac{y}{x}$

$$1+\left(\frac{y}{x}\right)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3 c \quad \text{——— } (11)$$

Put $x=2$, $y=6$ in eq (11)

$$4+36 = 8c$$

$$c = \frac{\cancel{40}^{\,5}}{\cancel{8}}$$

$$\boxed{c=5}$$

$\boxed{C=5}$ — put in eq (11)

$$x^2 + y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$y^2 = x^2(5x-1)$$

Taking square Root on both Side

$$y = +x\sqrt{\cancel{x}5x-1}, \quad y = -x\sqrt{5x-1}$$

$$\boxed{y = \pm x\sqrt{5x-1}}$$

Solution