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Subject :- Linear Algebra

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Question # 4

Part # A

Solutions

Let $m \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 2x2

$A = m^{-1} \frac{1}{det} m = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = da - bc$

entries and d for matrix m interchange put the negative error infront of b and c.

$B :- \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$

$= 9 - 8 = \boxed{1}$

So determinant is 1.

C:- $\begin{vmatrix} 2 & 2 \\ 3 & 3 \end{vmatrix}$

$= 6 - 6$

$= 0$

The determinant of matrix is 0

D:- $\begin{pmatrix} 1D1 & 1D1 & 1D1 \\ 1D2 & 1D3 & 1D2 \\ 1D4 & 1D1 & 1D5 \end{pmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 6 \\ 2 & 1 & 9 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 6 \\ 2 & 1 & 9 \end{bmatrix} - 1 \det \begin{bmatrix} 6 & 6 \\ 2 & 4 \end{bmatrix} + \det \begin{bmatrix} 6 & 7 \\ 2 & 1 \end{bmatrix}$

$= 1[63-6] - 1[54-12] + 1[14-6]$

$= 1[57] - 1[42] + 1[10]$

$= 25$ Answer

Question # 3

Part A:-

$V = (2\times2$ matrix with entries in R, usual matrix addition an

$$k \cdot \begin{pmatrix} a b \\ c d \end{pmatrix} = \begin{pmatrix} k\,a b \\ a c d \end{pmatrix} \text{ for } k \in R$$

$$\Rightarrow \quad k \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka_1 & kb_2 \\ kc_1 & kd_2 \end{bmatrix}$$

$= \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in R$ (taking $a' = ka_1 \in R =$ $b = kb_1$ $= kc_1$ and $d' = kd_1 \in R$)

Now

$$a' - 2b' = ka' - 2kb_1$$
$$= k\,(a_1 - 2b_1)$$
$$= k = 0$$
$$= 0$$

$$c' - 2d' = kc_1 - 2kd$$
$$= k\,(c_1 - 2d_1)$$
$$= k - 0$$
$$= 0$$

So $kA - \in R$

The implie that V ma satisfied $A_1 + A_2 \in V$ and kA $V, \in V$ where $k(A) \in R$ and $k \in R$ is any constant.

Question # 3

part B

Polyonomial of degree n does not form a vector space because don't form set closed under addition

For instance
$x^n - x^n = 0$
which is not of degree n

So don't get confused with the set of polynomials of degree less or equal than $n+1$ which form a vector space of dimension $n+1$ we often work with this space.

Question # 2

we want to prove $T(cv) = cT(v)$

$T(cv) = T(c[x_1, y_1, z_1])$
$= T([cx_1, cy_1, cz_1])$
$= [cx_1 + cy_1, cx_1 - cy_1, cz_1]$

and
$cT(v) = cT([x_1, y_1, z_1])$
$= c[x_1 + y_1, x_1 - y_1, z_1]$
$= [c(x_1 + y_1), c(x_1 - y_1), cz_1]$
$= [cx_1 + cy_1, cx_1 - cy_1, cz_1]$

so $T(cv) = cT(v)$

Question #1

sol

$v_1 \begin{bmatrix} 1 \\ 6 \\ 7 \end{bmatrix}, \quad v_2 \begin{bmatrix} 6 \\ 7 \\ 2 \end{bmatrix} \quad v_3 = \begin{bmatrix} 7 \\ 2 \\ 9 \end{bmatrix}$

Need to solve

$$\left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 6 & 7 & 2 & 0 \\ 7 & 2 & 9 & 0 \end{array}\right] R_2 + 1$$

$$\left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 7 & 8 & 3 & 0 \\ 7 & 2 & 9 & 0 \end{array}\right] = R_2 - R_3$$

$$\left[\begin{array}{ccc|c} 1 & 9 & 7 & 0 \\ 0 & 6 & -6 & 0 \\ 7 & 2 & 9 & 0 \end{array}\right] R_3 - 7 \left[\begin{array}{ccc|c} 1 & 9 & 7 & 0 \\ 0 & 6 & -6 & 0 \\ 0 & -5 & 2 & 0 \end{array}\right]$$

$R_2 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 0 & 6 & -6 & 0 \\ 0 & -5 & 2 & 0 \end{array}\right] R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & -5 & 2 & 0 \end{array}\right] R_3 + 2$$

$$\begin{bmatrix} 1 & 6 & 7 \\ 0 & 1 & -4 \\ 0 & -3 & 4 \end{bmatrix} R_2 + 3$$

$$\begin{bmatrix} 1 & 6 & 7 \\ 0 & 1 & -4 \\ 0 & -3 & 4 \end{bmatrix} \quad R_3 + R_2$$

$$= \begin{bmatrix} 1 & 6 & 7 \\ 0 & 1 & -4 \\ 0 & \cancel{-3} & 0 \end{bmatrix} \quad R_3 \times 4$$

$$= \begin{bmatrix} 1 & 6 & 7 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 6 & 7 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This show that matrix is infinite this is not lines independent.