Mid exam

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Section B

Subject Defferential equa

Semester 6th

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QN1

QNo1: Solve The initial value Problem

$= \frac{dy}{dt} = e^{y-t} \sec(y) (1+t^2)$ $y(0) = 0$

$\frac{dy}{dt} = \frac{e^{y} e^{-t} (1+t^2)}{\cos(y)}$

$e^{-y} \cos(y) dy = e^{-t} (1+t^2) dt$

integration by Part on both sides we can get an implicit soluation

$\int e^{-y} (\cos(y)) dy = \int e^{-t} (1+t^2) dt$

$\Rightarrow \frac{e^{-y}}{2} (\sin(y) - \cos(y)) = -e^{-t} (t^2+2t+3)$

$\Rightarrow \frac{1}{2}(-1) = -(3) + c \quad c = \frac{5}{2}$

There fore The implicit soluation

$\frac{e^{-y}}{2} (\sin(y) - \cos(y)) = -e^{t} (t^2+2t+3)$

Q No 2:. $(\sqrt{x+y} + \sqrt{x-y})dx - (\sqrt{x+y}) - \sqrt{x-y})dy = 0$

Soluation:. $(\sqrt{x+y}) + \sqrt{x-y})dx - (\sqrt{x+y}) - \sqrt{x-y})dy = 0$

$\Rightarrow (\sqrt{x+y}) + \sqrt{x-y})dx = (\sqrt{x+y}) - \sqrt{x-y})dy = 0$

$$\frac{dy}{dx} = \frac{(\sqrt{x+y}) + (\sqrt{x-y})}{(\sqrt{x+y}) - (\sqrt{x-y})}$$

$$\frac{dy}{dx} \Rightarrow \frac{\cancel{\sqrt{x}}(1+\frac{y}{\sqrt{x}}) + \cancel{\sqrt{x}}(1-y/\sqrt{x})}{\cancel{\sqrt{x}}(1+y/\sqrt{x}) - \cancel{\sqrt{x}}(1-y/\sqrt{x})}$$

$$\frac{dy}{dx} \Rightarrow \frac{(1+y/\sqrt{x}) + (1-y/\sqrt{x})}{(1+y/\sqrt{x}) - (1-y/\sqrt{x})}$$

Taking to square root of y

$$\frac{dy}{dx} = \frac{(1+\frac{\sqrt{y}}{\sqrt{x}}) + (1-\frac{\sqrt{y}}{\sqrt{x}})}{(1+\frac{\sqrt{y}}{\sqrt{x}}) - (1-\sqrt{y}/\sqrt{x})} \rightarrow (1)$$

$g(x) = \frac{\sqrt{y}}{\sqrt{x}}$ form

$V = \frac{y}{x}$ $\Rightarrow$ $y = Vx$

differential ~~with respective to~~ x
↳ w. r. t x

$$\frac{dy}{dx} = V\frac{dx}{dx} + x\frac{dV}{dx}$$

$$\frac{dy}{dx} = V + x\frac{dV}{dx}$$ put equation in (1)

$$V + x\frac{dV}{dx} = \frac{(1+\sqrt{V}) + (1-\sqrt{V})}{(1+\sqrt{V}) - (1-\sqrt{V})}$$

$$\frac{dy}{dx} = \frac{(1-\sqrt{V})^2}{1-(\sqrt{V})^2} \Rightarrow \frac{1-V}{1+V}$$

$$V + x\frac{dV}{dx} = \frac{1-V}{1+V} - V$$

$$x\frac{dV}{dx} = \frac{1-V-(1+V)}{1+V} \Rightarrow \frac{1-V-V+V^2}{1+V}$$

$$x\frac{dV}{dx} \Rightarrow \frac{1-2V+V^2}{1+V}$$

$$x\frac{dV}{dx} = \frac{V^2-2V+1}{1+V}$$

$$\frac{dV}{dx} = \frac{V^2-2V+1}{1+V}x$$

$$\int \frac{dx}{x} = \int \frac{1+v}{v^2-2v+1}$$

Multipling by 2 Namaratator of Right side

$$\Rightarrow \int \frac{dx}{x} = \int \frac{2v+2}{v^2+2v+1}$$

$$\cancel{\log} x = \cancel{\log}(v^2+2v+1)$$

$$x = v^2+2v+1$$

Put value of v

$$x = \left(\frac{y}{x}\right)^2 + 2\left(\frac{y}{x}\right) + 1$$

$$\boxed{x = \frac{y^2}{x^2} + \frac{2y}{x} + 1}$$

Q No 3:. $(D^4+D^2)y = 3x^2+4\sin x-2\cos x$

Soluation:. $(D^4+D^2)y = 3x^2+4\sin x-2\cos x$

$\Rightarrow F(D)\,y = f(x)$

As it is non- homogenous linear equation so soluation will be

$y = y_c + y_p$.

complementary soluation y_c

$D^4 - D^2 = 0 \Rightarrow D^2(D^2+1) = 0$

Either $D^2 = 0 \Rightarrow \boxed{D=0}$

$D^2+1=0 \Rightarrow D^2 = -1 \Rightarrow D^2+1=0 \Rightarrow D^2=-1$

$\Rightarrow D = \sqrt{-1} \Rightarrow D = i$ or $D = 0+i$

Roots are real and complex

$y_c = c_1e^{0x} + c_2\cos x + c_3\sin x)$

$y_c = c_1 + c_2\cos x + c_3\sin x$

$y_p = \frac{1}{f(D)} f(x)$

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$$y_p = \frac{1}{D^4+D^2}(3x^2+4\sin x-2\cos x)$$

$$\Rightarrow \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$$\Rightarrow f(D) = D^4+D^2$$

at $D=0$ $\quad F(D)=0$

so $f'(D) = 4D^3+2D$

Now also for $D=0 \Rightarrow F'(D)=0$
again differentiating

$$F''(D) = 12D+2$$
for $D=0$

$$F''(0) = 12(0)+2 = 2$$

so replacing $\frac{1}{F(D)}$ with $\frac{x^2}{F''(D)}$

$$\Rightarrow y_p = \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2}\cdot 4\sin x - \frac{x^2}{12D+2}\cdot 2\cos x$$

Putting $D=0$ in all

$$y_p = \frac{x^2\cdot 3x^2}{12(0)+2} + \frac{x^2 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}$$

P.T.O

$$y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2 \cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2 \sin x - x^2 \cos x$$

so Putting in equation (i)

$$y = c_1 + c_2 \cos x + c_3 \cos x + \tfrac{3}{2}x^2 + 2x^2 \sin x$$
$$- x^2 \cos x$$

$$y = c_1 + (c_2 - x^2)\cos x + (c_3 + 2x^2)\sin x + \tfrac{3}{2}x^4$$