IQRA NATIONAL UNIVERSITY

FINAL ASSIGNMENT BS SOFTWARE ENGINEERING

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SEMESTER 2nd BS(SE)

SUBJECT: DISCRETE STRUCTURES

Q.1

Explain the concept of Biconditional statement

Answer:

A biconditional statement is a combination of a conditional statement and its converse written in the if and only if form.

Two line segments are congruent if and only if they are of equal length.

It is a combination of two conditional statements, “if two line segments are congruent then they are of equal length” and “if two line segments are of equal length then they are congruent”.

A biconditional is true if and only if both the conditionals are true.

Bi-conditionals are represented by the symbol $\leftrightarrow\leftrightarrow$ or $\Leftrightarrow\Leftrightarrow$.

$p\leftrightarrow qp\leftrightarrow q$ means that $p\rightarrow qp\rightarrow q$ and $q\rightarrow pq\rightarrow p$. That is, $p\leftrightarrow q=(p\rightarrow q)\wedge(q\rightarrow p)p\leftrightarrow q=(p\rightarrow q)\wedge(q\rightarrow p)$.

Example:

Write the two conditional statements associated with the bi-conditional statement below.

A rectangle is a square if and only if the adjacent sides are congruent.

The associated conditional statements are:

a) If the adjacent sides of a rectangle are congruent then it is a square.

b) If a rectangle is a square then the adjacent sides are congruent.

Truth table for bi-conditional:

P	Q	P↔Q
T	T	T
T	F	F
F	T	F
F	F	T

It is to be noted that $p\leftrightarrow q$ has exactly the same truth value as $(p\text{->} q) \wedge (q\rightarrow p)$

P	Q	p↔q	q→p	p→q	(p→q)∧(q→p)
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

b) Let p, q, and r represent the following statements:

p: Sam had pizza last night.

q: Chris finished her homework.

r: Pat watched the news this morning

Give a formula (using appropriate symbols) for each of these statements.

i. Sam had pizza last night if and only if Chris finished her homework.

ii. Pat watched the news this morning if Sam did not have pizza last night.

iii. Pat watched the news this morning if and only if Chris finished her homework and Sam did not have pizza last night.

iv. In order for Pat to watch the news this morning, it is necessary and sufficient that Sam had pizza last night and Chris finished her homework

ANSWER:

a) Sam had pizza last night if and only if Chris finished her homework.

$p \Leftrightarrow q$

b) Pat watched the news this morning if Sam did not have pizza last night.

$r \Leftrightarrow \neg p$

c) Pat watched the news this morning if and only if Chris finished her homework and Sam did not have pizza last night.

$r \Leftrightarrow (q \wedge \neg p)$

d) In order for Pat to watch the news this morning, it is necessary and sufficient that Sam had pizza last night and Chris finished her homework.

$r \Leftrightarrow (p \wedge q)$

e) $q \Leftrightarrow r$

Chris finished his homework if and only if Pat watched the news this morning

f) $p \Leftrightarrow (q \wedge r)$

Sam had pizza last night if and only if Chris finished his homework and Pat watched the news this morning

g) $(\neg p) \Leftrightarrow (q \lor r)$

Sam didn't have pizza last night if and only if Chris finished his homework or Pat watched the news this morning

h) $r \Leftrightarrow (p \lor q)$

Pat watched the news this morning if Sam had pizza last night or Chris finished his homework

Question 2: Lets p, q, r represent the following statements:

p: it is hot today.

q: it is sunny

r: it is raining

Express in words the statements using Bicondtional statement represented by the following formulas:

i. $q \leftrightarrow p$
ii. $p \leftrightarrow (q \wedge r)$
iii. $p \leftrightarrow (q \vee r)$
iv. $r \leftrightarrow (p \vee q)$

Answers:

i. it is sunny if and only if it is hot today (true)
ii. it is hot today if and only if it is sunny and it is raining(false)
iii. it is hot today if and only if it is it is sunny or it is raining (true)
iv. it is raining if and only if it is hot today or it is sunny (false)

Question 3:Explain Argument with proper examples. Differentiate Valid and Invalid argument through proper examples, also construct a truth table showing valid and invalid arguments. (Note: Examples and truth table should not belongs to your book or slides)

Answer:

Argument:

An argument is a rationale in which the reason presents evidence in support of a claim made in the conclusion. Its purpose is to provide a basis for believing the conclusion to be true. It can also be defined as a sequence of propositions called premises followed by a proposition called conclusion. A valid argument is one that, if all its premises are true, then the conclusion is true. Example of an argument can be:

"If it rains, I drive to school." "It rains."

"I drive to school."

Difference between valid invalid arguments:

Valid: an argument is valid if and only if it is necessary that if all of the premises are true, then the conclusion is true; if all the premises are true, then the conclusion must be true; it is impossible that all the premises are true and the conclusion is false.

Invalid: an argument that is not valid. We can test for invalidity by assuming that all the premises are true and seeing whether it is still possible for the conclusion to be false. If this is possible, the argument is invalid.

Validity and invalidity apply only to arguments, not statements. For our purposes, it is just nonsense to call a statement valid or invalid. True and false apply only to statements, not arguments. For our purposes, it is just nonsense to call an argument true or false. All deductive arguments aspire to validity.

For Example:

If you consider the definitions of validity and invalidity carefully, you'll note that valid arguments have the following important property: valid arguments preserve truth. If all your premises are true and you make a valid argument from them, it must be the case that whatever conclusion you obtain is true. (We shall see below, however, that valid arguments do not necessarily preserve truth value: it is entirely possible to argue validly from false premises to a true conclusion).

Sound: an argument is sound if and only if it is valid and contains only true premises.

Unsound: an argument that is not sound.

Counterexample: an example which contradicts some statement or argument (ex. a counterexample to the statement "All fifteen year-olds have blue hair" would be a fifteen-year-old without blue hair); for an argument, a counterexample would be a situation in which the premises of the argument are true and the conclusion is false; counterexamples show statements to be false and arguments to be invalid.

Truth table for valid and invalid arguments:

A	B	A ⊃ B	~B	∴ ~A
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Since no row has all true premises and a false conclusion the argument is valid. The only row with all true premises is the bottom row and on that row the conclusion is true. The important thing is not that there's a row with all true premises and a true conclusion but rather that there's no row with all true premises and a false conclusion.

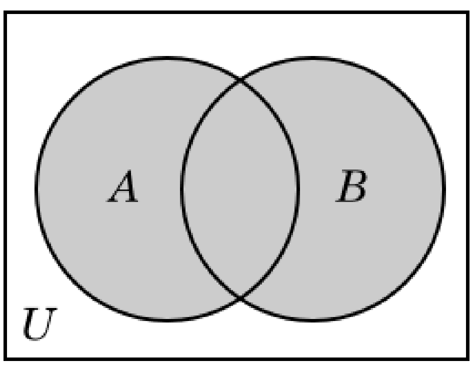
Question 4:

Explain the concept of Union, also explain membership table for union by giving proper example of truth table.

Answer:

Union:

In set of theory, the union denoted by U of a collection of sets is the set of all elements in the collection it is one of the fundamental operations through which sets can be combined and related to each other



Let A & B are subsets of universal sets U

The union of A & B is the set of all elements in U that belong to A or to B or both

It is denoted by A ∪ B

A ∪ B= {x ∈U | x ∈A or x ∈ B}

Union is commutative: A ∪ B = B ∪ A

A ⊆ A ∪ B and B ⊆ A ∪ B

Example: Let U = {10,11,12,13,14,15,16,17 } A = {10, 12, 14, 17}, B = {13,14, 15,16}
Then A ∪ B = {10,11,12, 13, 14, 15, 16,17}

Membership Table for Union:

- The Membership table for the union of sets A and B is given below

- The truth table for disjunction of two statements P and Q is given below
- In the membership table of Union replace, 1 by T and 0 by F then the table is same as of disjunction
- So membership table for Union is similar to the truth table for disjunction

A	B	A ∪ B
1	1	1
1	0	1
0	1	1
0	0	0

P	Q	P v Q
T	F	T
T	F	T
F	T	T
F	F	F

b) Explain the concept of Intersection, also explain membership table for Intersection by using proper example of truth table

Intersection:

The intersection of two sets A & B denoted by A ∩ B is the set containing all elements of A that also belongs to B or all elements of B that also belong to A

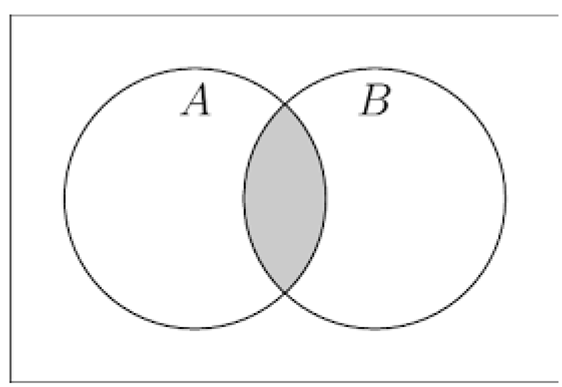
Intersection is written using the sign ∩

$A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}$

Intersection is commutative: $A \cap B = B \cap A$

$A \cap B \subseteq A$ and $A \cap B \subseteq B$

If A and B are disjoint, then $A \cap B = \phi$



Example: Let U = {a, b, c, d, e, f, g}

A = {a, c, e, g}, B = {d, e, f, g}

Then A ∩ B = {e, g}

Membership Table For Intersection:

- The Membership table for intersection of sets A and B is given
- below
- The truth table for conjunction of two statements P and Q is given

- below
- In the membership table of Intersection, replace 1 by T and 0 by F
- then the table is same as of conjunction
- So membership table for Intersection is similar to the truth table
- for conjunction (∧)

A	**B**	**A ∩ B**
1	1	1
1	0	0
0	1	0
0	0	0

P	**Q**	**P ∧ Q**
T	F	T
T	F	T
F	T	T
F	F	F

Question 5:

a) Explain the concept of Venn diagram with examples.

b) Given the set *P* is the set of even numbers between 15 and 25. Draw and label a Venn diagram to represent the set *P* and indicate all the elements of set *P* in the Venn diagram.

c) Draw and label a Venn diagram to represent the set

R = {Monday, Tuesday, Wednesday}.

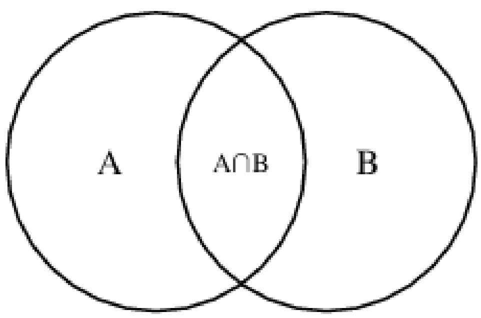
d) Given the set $Q = \{x : 2x - 3 < 11, x$ is a positive integer $\}$. Draw and label a Venn diagram to represent the set Q.

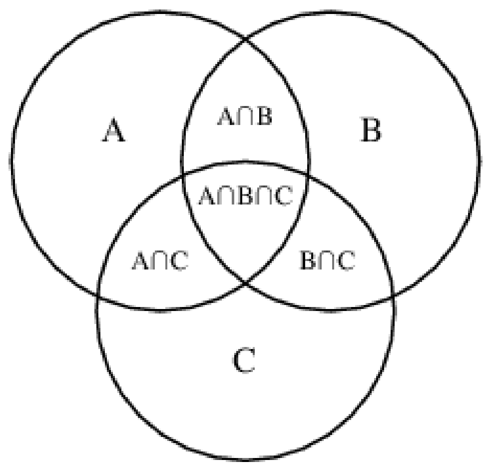
ANSWER:

a) Explain the concept of Venn diagram with examples.

A Venn diagram is an illustration of the relationships between and among sets, groups of objects that share something in common. Usually, Venn diagrams are used to depict set intersections (denoted by an upside-down letter U). This type of diagram is used in scientific and engineering presentations, in theoretical mathematics, in computer applications, and in statistics. The drawing is an example of a Venn diagram that shows the relationship among three overlapping sets X, Y, and Z. The intersection relation is defined as the equivalent of the logic

AND. An element is a member of the intersection of two sets if and only if that element is a member of both sets. Venn diagrams are generally drawn within a large rectangle that denotes the universe, the set of all elements under consideration. In this example, points that belong to none of the sets X, Y, or Z are gray. Points belonging only to set X are cyan in color; points belonging only to set Y are magenta; points belonging only to set Z are yellow. Points belonging to X and Y but not to Z are blue; points belonging to Y and Z but not to X are red; points belonging to X and Z but not to Y are green. Points contained in all three sets are black. Here is a practical example of how a Venn diagram can illustrate a situation. Let the universe be the set of all computers in the world. Let X represents the set of all notebook computers in the world. Let Y represent the set of all computers in the world that are connected to the Internet. Let Z represent the set of all computers in the world that have anti-virus software installed. If you have a notebook computer and surf the net, but you are not worried about viruses, your computer is probably represented by a point in the blue region. If you get concerned about computer viruses and install an anti-virus program, the point representing your computer will move into the black area.





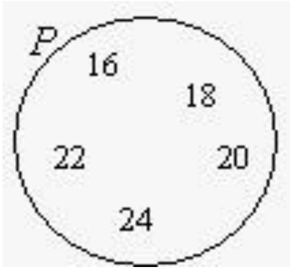
b) Given the set *P* is the set of even numbers between 15 and 25. Draw and label a Venn diagram to represent the set *P* and indicate all the elements of set *P* in the Venn diagram.

Solution:

List out the elements of P.

P = {16, 18, 20, 22, 24} ← ‘between’ does not include 15 and 25

Draw a circle or oval. Label it P. Put the elements in P

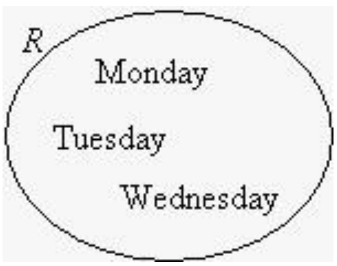


c) Draw and label a Venn diagram to represent the set

R **= {Monday, Tuesday, Wednesday}.**

Solution:

Draw a circle or oval. Label it R. Put the elements in R



d) Given the set *Q* = {*x* : 2*x* – 3 < 11, *x* is a positive integer }. Draw and label a Venn diagram to represent the set *Q*.

Solution:

Since an equation is given, we need to first solve for x.

$2x - 3 < 11 \Rightarrow 2x < 14 \Rightarrow x < 7$

So, Q = {1, 2, 3, 4, 5, 6}