Assignment

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Section A

Subject Numerical analysis

Submitted to

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Question No 1

ANSWER:

Find the root of the equation

$$x^3 + 3.993 \times 10^{-4} = 0.16x^2$$

Solution:-

use newton raphson method with

$$x_0 = 0.02$$

Rearranging the equation

$$x^3 - 0.165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 0.33x = 0$$

$$\rightarrow \quad x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$\therefore f(x_0) = 0.00033$

$\therefore f''(x_0) = -0.0054$

$$= 0.02 - \frac{0.00033}{-0.0054}$$

$$\boxed{x_1 = 0.081}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$\because f(x_1) = -0.00016$

$\because f'(x_1) = -0.0070$

$$x_2 = 0.081 - \frac{(-0.00016)}{-0.0070}$$

$$\boxed{x_2 = 0.058}$$

$$\rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$\because f(x_2) = 0.000030$

$\because f'(x_2) = -0.0090$

$$x_3 = 0.058 - \frac{0.000030}{(-0.0090)}$$

$$\boxed{x_3 = 0.061}$$

Question NO 2

ANSWER:-

use the number $x_0 = 2$, $x_1 = 2.75$, $x_2 = 4$ find the Lagrang Interpolation polynomial for $(x)^{1/x}$ $x = 3$

Solution:-

$x_0 = 2$, $y_0 = 0.5$

$x_1 = 2.75$, $y_1 = 0.36$

$x_2 = 4$, $y_2 = 0.25$

As we know that

Lagrange Interpolation formula

$$y = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} y_0$$

$x_0 = 2$ | $y = 0.5$

$x_1 = 2.75$ | $y_1 = 0.36$

$x_2 = 4$ | $y_2 = 0.25$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x-x_1)} y_2$$

Putting value

$$y = \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)} (0.5)$$

$$+ \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)} (0.36)$$

$$+ \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)} (0.25)$$

$$y = (-0.833) + 0.384 + 0.025$$

$$\boxed{y = -0.424}$$

Question NO3

ANSINER:-

Complete the divided differences table for the given data and construct the Interpolating polynomial that uses all this data.

x	1.0	1.3	1.6	1.9	2.2
y	0.7651977	0.6200860	0.4555402	0.2818186	0.1103623

Solution:-

x_i	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i-1}, x_i)$	$f(x_{i-3}, \dots, x_i)$	$f(x_{i-4}, x_{i-3}, x_{i-2}, x_{i-1}, x_i)$
x_0 1.0	0.7651977				
		−0.4837056			
x_1 1.3	0.6200860		−0.108734		
		−0.548946		0.0658785	
x_2 1.6	0.4555402		−0.0444433		−0.0023049
		−0.578672		0.0625255	
x_3 1.9	0.2818186		0.006818		
		−0.571521			
x_4 2.2	0.1103623				

i) $f(x_0, x_1) = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \dfrac{0.6200860 - 0.7651977}{1.3 - 1}$

$\boxed{f(x_0, x_1) = -0.4837056}$

ii) $f(x_1, x_2) = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{0.4554022 - 0.6200860}{1.6 - 1.3}$

$\boxed{f(x_1, x_2) = -0.548946}$

iii) $f(x_2, x_3) = \dfrac{f(x_3) - f(x_2)}{x_3 - x_2}$

$= \dfrac{0.2818186 - 0.4554022}{1.9 - 1.6}$

$\boxed{f(x_2, x_3) = 0.578612}$

iv) $f(x_2, x_3) = \dfrac{f(x_4) - f(x_3)}{x_4 - x_3}$

$= \dfrac{0.1103623 - 0.2818186}{2.2 - 1.9}$

$\boxed{f(x_3, x_4) = -0.571521}$

Second divided difference :-

$$f(x_0,x_1,x_2) = \frac{f(x_1,x_2) - f(x_0,x_1)}{x_2 - x_0}$$

$$= \frac{-0.548946 - (-0.4837056)}{1.6 - 1}$$

$$\boxed{f(x_0,x_1,x_2) = 0.108734}$$

$$f(x_1,x_2,x_3) = \frac{f(x_2,x_3) - f(x_1,x_2)}{x_3 - x_1}$$

$$= \frac{-0.578612 - (-0.548946)}{1.9 - 1.03} \Rightarrow \boxed{f(x_1,x_2,x_3) = -0.0494433}$$

$$f(x_2,x_3,x_4) = \frac{f(x_3,x_4) - f(x_2,x_3)}{x_4 - x_2}$$

$$= \frac{-0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$\boxed{f(x_2,x_3,x_4) = 0.006818}$$

Third divided differences

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{-0.0494433 - (-0.108734)}{1.9 - 1}$$

$$\boxed{f(x_0, x_1, x_2, x_3) = 0.0658785}$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.006818 - (-.049443)}{2.2 - 1.3}$$

$$\boxed{= 0.06251255}$$

4th divide difference..

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{0.06251255 - 0.0658785}{2.2 - 1}$$

$$\boxed{= 0.0028049}$$

$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2)$$
$$+ (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3) + (x - x_0)(x - x_1)$$
$$(x - x_2)(x - x_3) f(x_0, x_1, x_2, x_3, x_4)$$

$$= 0.7651977 + (x - 1) - 0.4837056 + (x - 1)(x - 1.3)(-0.108734)$$
$$+ (x - 1)(x - 1.3)(x - 1.6)(0.0658785) + (x - 1)(x - 1.3)$$
$$(x - 1.6)(x - 1.9)(-0.0028049)$$

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