Mid Term Assignment

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Subject : Multi Variate calculus

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Mid paper
9th Semister.

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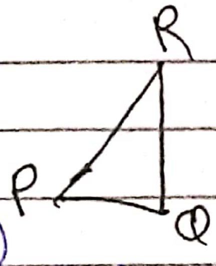
Q1) Let $P(1,0,-3)$, $Q(0,-2,-4)$ and $R(4,1,6)$ be points.

a) find the equations of the plane through the points P, Q and R.




**Solution:-**

a) $\vec{PQ}\,(0-1, -2-0, -4-(-3))$
$= (-1,-7,-1)$

$\vec{PR} = (4-1,\ 1-0,\ 6-(-3)) = (3,1,9)$

now, Equation of plane is;

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

where $(x_0, y_0, z_0)$ can be any point, and $(a,b,c)$ is any line perpendicular to

$P \cdot T = 0$

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the plane of the point.

which means

$(a, b, c) = \overrightarrow{PQ} \times \overrightarrow{PR}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & -1 \\ 3 & 1 & 9 \end{vmatrix}$$

$$\Rightarrow \hat{I} \begin{vmatrix} -2 & -1 \\ 1 & 9 \end{vmatrix} + \hat{j} \begin{vmatrix} -1 & -1 \\ 3 & 9 \end{vmatrix} + \hat{k} \begin{vmatrix} -1 & -2 \\ 3 & 1 \end{vmatrix}$$

$(a, b, c) = -17\hat{i} - 6\hat{j} + 7\hat{k}$

$\Rightarrow (-17, -6, 7)$

Equation

$-17(x-1) - 6(y-0) + 7(z+$

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$\Rightarrow -17x + 17 - 6y + 7z + 21 = 0$

$17x + 6y - 7z = 38$ Ans

Q1) find the area of the
b) triangle with vertices
P, Q and R.

Sol/
$P(1, 0, -3), Q(0, -2, 4)$ $R(4, 1, 6)$

$V_1 = PQ = (0-1)(-2, -0)(-4-3)$

$V_1 = PQ = -1, -2, -7$

$V_2 = PR = (4, -1)(1-0)(6-(-3))$

$V_2 = PR = 3, 1, 9$

Now

$V1 \times V2$

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$V_2 = PR = 3, 1, 9$

Now

$V1 \times V2$

$$\begin{vmatrix} i & j & k \\ -1 & -2 & -7 \\ 3 & 1 & 9 \end{vmatrix}$$

$= -11i - 12j + 5k$

Area of triangle

$A = \frac{1}{2} \| v_1 \times v_2 \|$

$= \frac{1}{2} \sqrt{(-11)^2 + (12)^2 + (5)^2}$

$= \frac{1}{2} \sqrt{121 + 144 + 25}$

$= \frac{1}{2} \sqrt{290}$

$\boxed{A = 8.51}$ Ans

Q2) find the distance between the parallel planes $x + 2y - z = -1$ and $3x + 6y - 3z = 3$ use the following formula to find the distance btw the given parallel plane's.

[illegible]

Solution

Point distance formula

$$d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

P. (-0)

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$x + 2y - z + 1 = 0 \longrightarrow eq(i)$

$3x + 6y - 3z - 3 = 0 \longrightarrow eq(ii)$

$\frac{1}{3} = \frac{2}{6} = \frac{-1}{-3} = \frac{1}{3}$ (Parallel)

form(i) $= a = 1, b = 2, c = -1, d = 1$

form(ii) $x = 0, y = 0$

$$-3z = 3$$

$$z = -1$$

$$d = \left| \frac{1(0) + 2(0) - 1(-1) + 1}{\sqrt{1^2 + 2^2 + (-1)^2}} \right|$$

$$d = \frac{2}{\sqrt{6}}$$ Ans

3) (Q3) find the following limit, if it exists, or show that the limit not exist

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - xy + y^2}{x^2 + y^2}$$

Solution:-

Along x-axis, $y = 0$

$$\lim_{(x,y)\to(0,0)} \frac{\cancel{x^2}}{\cancel{x^2}} = 1$$

Along y-axis, $x = 0$

$$\lim_{(x,y)\to(0,0)} \frac{\cancel{y^2}}{\cancel{y^2}} = 1$$

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Along $y = x$,

$$\frac{x^2 - x^2 + x^2}{x^2 + x^2} = \frac{x^2}{2x^2} = \frac{1}{2}$$

$$\lim_{(x,y) \to (0,0)}$$

limit does not exist.

Q9) find the directional derivative of the function $f(x,y,z) = xyz$ in the direction of vector $v = \langle 5, -3, 2 \rangle$

Solution $f(x,y,z) = xyz$

$$\frac{\partial f}{\partial x} = yz$$

$$\frac{\partial f}{\partial y} = xz$$

$$\frac{\partial f}{\partial z} = xy$$

$\nabla f(x,y,z) = (yz, xz, xy)$

$$\hat{V} = \frac{-1, -2, 12}{\sqrt{1+4+144}} = \frac{1}{\sqrt{149}}(-1, -2, 12)$$

Directional derivative $= \nabla f(x,y,z) \cdot \hat{V}$

P.T.O

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Q5) find the equation of the tangent plane to the surface $z = 4x^3y^2 + 2y$ at point $(1, -2, 12)$

Solution

Eq of tangent plane to the surface $z = f(x, y)$ at the point $(a, b, f(a, b))$

$$\frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b) - z$$

$$+ f(a,b) = 0$$

$z = 4x^3y^2 + 2y$, $(1, -2, 12)$

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$$\frac{\partial z}{\partial x} = 12x^2y^2 \qquad \frac{\partial z}{\partial y} = 8x^3y + 2$$

$$\left.\frac{\partial z}{\partial x}\right|_{(1,-2)} = 48 \qquad \left.\frac{\partial z}{\partial y}\right|_{(1,-2)} = -14$$

$$48(x-1) - 14(y+2) - z - 12 = 0$$

$$z = 4(1)^3(-2)^3(-2)^2 + 2(-2)$$

$$z = 4(4) - 4$$

$$z = 16 - 14 = 12$$

putting in eq

$$48(x-1) - 14(y+2) - z - 12 = 0$$

$$48x - 48 - 14y - 28 - 12 - 12 = 0$$

$$48x - 14y = 48 + 28 + 12 + 12$$

$$48x - 14y = 90 \text{ Ans}$$

Q6) Let $u = \langle u_1, u_2, u_3 \rangle$ and $v = \langle v_1, v_2, v_3 \rangle$ be any two vector's in space. Show the following identity that relate's the cross product and the dot product $|u \times v|^2 + |u \cdot v|^2 = |u|^2 |v|^2$

Solution

$$u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$u \cdot v = u_1 v_1 + u_2 v_2 + u_2 v_3$

$$u \times v = i(u_2v_3 - v_2u_2) - j(u_1v_3 - v_1u_3)$$
$$+ k(u_1v_2 - v_1u_2)$$

$$u \times v = i(u_2v_3 - v_2u_3) + j(v_1u_3 - u_1v_3)$$
$$+ k(u_1v_2 - v_1u_2)$$

$$|u \times v|^2 = (v_2v_3 - v_2u_3)^2 + (v_1u_3 - u_1v_3)^2$$
$$+ (u_1v_2 - v_1u_2)^3$$

$$|u \cdot v|^2 = (u_1v_1 + u_2v_3 + u_3v_3)^2$$
$$= (u_1^2 + u_2^3 + u_3^2)(v_1^2 + v_2^2 + v_3^2)$$

Q7) What is the angle between the two planes $x+y=0$ and $y-z=2$?

Solution

$x+y=0$

$x_1+y_2+0z_1=0 \rightarrow$ Eq (1)

$y-z=2$

$0x_2+y_2-z_2=2 \rightarrow$ Eq (2)

Angle btw plane formula

$$\cos\theta = \frac{(x_1x_2+y_1y_2+z_1z_2)}{\sqrt{x_1^2+y_1^2+z_1^2}\,,\sqrt{0^2+1^2+(-1)^2}}$$

$$\cos\theta = \frac{1}{\sqrt{2}\;\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3} = 60^\circ$$