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Subject	Differential equation
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Q.1.

Solution:-

$$f(t) = 1 + t \qquad -\pi \leq t \leq \pi$$

Here we use the formula

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1} b_n \sin t \longrightarrow eq(1)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\,dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\,dt$$

$$a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2}\right) \right)$$

$$a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi^2}{2} \right)$$

$$\boxed{a_0 = \frac{1}{2\pi} \left(2\pi + \pi^2 \right)}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)(\cos nt)\,dt$$

$$a_n = \frac{1}{\pi}\left(\left.(1+t)\,\frac{\sin nt}{n}\right|_{-\pi}^{\pi} - \int\left(\frac{\sin nt}{n}\,\frac{d}{dt}(1+t)\right)\right)$$

$$a_n = \frac{1}{\pi}\left(\left.(1+t)\,\frac{\sin \cancel{n}t}{\cancel{n}} - \frac{\cos nt}{n^2}\right|_{-\pi}^{\pi}\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(-1-(-1)\right)$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)\sin nt\,dt$$

$$b_n = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi} \sin nt - \int\left(\int \sin nt\,\frac{d}{dt}(1+t)\,dt\right)\right)$$

$$b_n = \frac{1}{\pi}\left(\left.\frac{(1+t)(-\cos nt)}{n}\right|_{-\pi}^{\pi} - \int\left(\frac{-\cos nt}{n}(1)\right)\right)$$

$$b_n = \frac{1}{\pi}\left(\left.-\frac{(1+t)(\cos nt)}{n}\right|_{-\pi}^{\pi} + \left(\left.\frac{\sin \cancel{n}\pi}{n^2}\right|_{-\pi}^{\pi}\right)\right)$$

$$b_n = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - \left((1+(\pi)(\cos n(x)\right)\right)$$

$$b_n = \frac{-1}{n\pi}\left(\cos \cancel{n}\pi + \pi \cos n\pi - \cos \cancel{n}\pi + \pi \cos n\pi\right)$$

$$b_n = \frac{-1}{n\cancel{\pi}}\left(2\cancel{\pi} \cos n\pi\right)$$

~~Ans~~

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$$b_n = \frac{2}{\pi}(-1)^{n+1}$$

So eqn become

$$\boxed{f(x) = \frac{1}{2\pi}\left(2\pi + \pi^2\right) + 0 + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx}$$

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Q2.

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen values = ?

Solution:-

Step # 1.

We have

$(A - \lambda I)x = 0$ $\quad$ A = Given matrix

Step # 2

We have; The characteristic eq is given by

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{bmatrix} = 0$$

Step #3

$$\lambda^3 - \left|\text{sum of Diognal elem}\right| \lambda^2 + \left|\text{sum of Diognal minor}\right| \lambda - |A| = 0 \quad \text{— (B)}$$

Sum of Diognal elements $= 1+1+2 = 4$

Sum of Diognal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (1)$

$= -6+2+1$

$= 3$

By putting values in eq (B)

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{— (C)}$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 1(2-8) - 0 + 1(6-0)$

$= 0$

By putting in eq (C)

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0$$

$$\lambda = 0$$

$$\lambda^2 - 4\lambda - 3 = 0$$

Using Quadratic formula

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$

$b = -4$

$c = -3$

$$= \frac{(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 12}}{2}$$

$$= \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4 + \sqrt{28}}{2}, \quad \lambda = \frac{4 - \sqrt{28}}{2}$$

We have eigen values;

$$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right)$$

Ans

Q3. Solve the following system of linear equation.

$$
\begin{aligned}
5x \quad\quad & +4z \; +2m = 3 \\
x - y \; & +2z \; + m = 1 \\
4x + y \; & +2z \quad\quad\; = 1 \\
x + y \; & + z \; + m = 0
\end{aligned}
$$

Solution:-

Writing in matrix form

$$
\begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}
\begin{bmatrix} x \\ y \\ z \\ m \end{bmatrix}
=
\begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix}
$$

The Augmented Matrix "Ab" is

$$
\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right]
$$

We solve the given system by Gauss elimination method.

$$R_1 \leftrightarrow R_4 \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right]$$

$$R_2 - R_1 \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right]$$

$$R_3 - 4R_1 \sim \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right]$$

$$R_4 - 5R_1 \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right]$$

$$R_2 \div (-2) \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1/2 & 0 & -1/2 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & 3 & 3 \end{array}\right]$$

$$R_3 + 3R_2 \quad \sim \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & -\frac{7}{2} & -4 & -\frac{1}{2} \\ 0 & -5 & -1 & -3 & 3 \end{array}\right]$$

$$R_4 + 5R_2 \quad \sim \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & -\frac{7}{2} & -4 & -\frac{1}{2} \\ 0 & 0 & -\frac{7}{2} & -3 & \frac{1}{2} \end{array}\right]$$

$$R_4 + \tfrac{7}{2} R_3 \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{8}{7} & \frac{1}{7} \\ 0 & 0 & -\frac{7}{2} & -3 & \frac{1}{2} \end{array}\right]$$

Using backward substitutions

$$0.x + 0.y + 0.z + 1m = 1$$

$$\Rightarrow \boxed{m = 1}$$

$$0.x + 0.y + 1.z + \tfrac{8}{7}m = \tfrac{1}{7}$$

$$z = \tfrac{8}{7}(1) = \tfrac{1}{7}$$

$z = 1/7 - 8/7$

$z = \frac{1-8}{7}$

$z = -7/7$

$\boxed{z = -1}$

$0.x + 1.y - 1/2\ z + 0.m = -1/2$

$y - 1/2\ z = -1/2$

$y - 1/2(-1) = -1/2$

$y + 1/2 - 1/2$

$y = -1/2 - 1/2$

$y = \frac{-1-1}{2}$

$y = -2/2$

$\boxed{y = -1}$

$1 \cdot x + 1 \cdot y + 1 \cdot z + 1 \cdot m = 0$

$x + (-1) + (-1) + 1 = 0$

$x - 1 - 1 + 1 = 0$

$\boxed{x = 1}$

So,

$x = 1$, $y = -1$, $z = -1$ and $m = 1$

is the solution.

Q 4 verify that

$$u(x,t) = \sin(x+2t)$$

is a solution of the one-dimensional equation.

Solution:-

Given that

$$u(x,t) = \sin(x+2t)$$

Differentiate w.r.t x partialy

$$\frac{du}{dx} = \frac{d}{dx}\sin(x+2t)$$

$$\frac{du}{dx} = \cos(x+2t)\frac{d}{dx}(x+2t)$$

$$\frac{du}{dx} = \cos(x+2t)(1+0)$$

$$\frac{du}{dx} = \cos(x+2t)$$

$$\frac{d^2u}{dx^2} = \frac{d}{dx}\cos(x+2t)$$

$$\frac{d^2u}{dx^2} = -\sin(x+2t) \cdot \frac{d}{dx}(x+2t)$$

$$\frac{d^2u}{dx^2} = -\sin(x+2t)(1+0)$$

$$\frac{d^2u}{dx^2} = -\sin(x+2t)$$

and $u(x,t) = \sin(x+2t)$

Differentiate w.r.t "t"

$$\frac{du}{dt} = \frac{d}{dt}\sin(x+2t)$$

$$\frac{du}{dt} = \cos(x+2t) \quad (0+2)$$

$$\frac{du}{dt} = 2\cos(x+2t)$$

$$\frac{d^2u}{dt^2} = (2) - \sin(x+2t) \quad (0+2)$$

$$\boxed{\frac{d^2u}{dt^2} = -4\sin(x+2t)}$$

We know that one-dimensional evuation is

$$\frac{d^2u}{dt^2} = c^2 \frac{d^2u}{dx^2}$$

$$-4\sin(x+2t) = c^2[-\sin(x+2t)]$$

$$-4\sin(x+2t) = -c^2 \sin(x+2t)$$

$$-4\sin(x+2t) + c^2 \sin(x+2t) = 0$$

For the orbitrary constant $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

Then it will verified for the orbitrary constant $c = 2$.