Mid term
Exam Summer

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Section:- B

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Dept:- BE(CE)

Subject:- Differential equation

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Question : 1

$$\frac{dy}{dt} = e^{y-t} \sec(y)(1+t^2) \qquad y(0)=0$$

By seperaing Variables

$\therefore \cos = 1/\sec$

$$\int e^{-y} \cos y \, dx = \int (1+t^2) e^{-t} dt$$

integrating by parts (I)

$u = e^{-y}$ $\qquad dv = \cos y \, dy$

$du = -e^{-y} dy$ $\qquad V = \sin y$

integrating by parts (II)

②

$u = e^{-y}$ $dv = \sin y\, dy$

$du = -e^{-y}\, dy$ $V = -\cos y$

by integrating 1st part

LHS $= e^{-y}\sin y + \int e^{-y}\sin y\, dy$

by integrating part 2

$= e^{-y}\sin y - e^{-y}\cos y - \int e^{-y}\cos y\, dy$

Since the last integral is came as L.H.S

L.H.S $= e^{-y}(\sin y - \cos y)$

By adding L.H.S

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$2\,(LHS) = e^{-y}(\sin y - \cos y)$

$\div$ ing by 2

$L.H.S = \frac{e^{-y}}{2}(\sin y - \cos y)$

integrating part (3)

$v = 1+t^2 \qquad du = e^{-t}\,dt$

$du = 2t\,dt \qquad v = e^{-t}$

integrating by part (4)

$u = 2t \qquad dv = e^{-t}\,dt$

$du = 2\,dt \qquad v = -e^{-t}$

Let us evaluate R.H.S

by integrating part 3

R.H.S $= -(1+t^2)e^{-t} + \int 2te^{-t}\,dt$

by integrating part 4

$= -(1+t^2) - 2te^{-t} + \int 2e^{-t}\,dt$

$= -(t^2+2t+1)e^{-t} - 2e^{-t} + C$

$= -(t^2+2t+3)e^{-t} + C$

Comparing L.H.S and R.H.S

(5)

$$\frac{e^{-y}}{2}(\sin y - \cos y) = -(t^2+2t+3)e^{-t} c$$

Since $y(0)=0$ we have

$$\frac{1}{2}(0-1) = -3+c$$

$$c = 5/2$$

Hence the solution is implicity explessed as

$$\frac{e^{-y}}{2}(\sin y - \cos y) = -(t^2+2t+3)e^{-t} + 5/2$$

Question - 2

$$\left(\sqrt{x+y}+\sqrt{x-y}\right)dx-\left(\sqrt{x+y}-\sqrt{x-y}\right)dy=0$$

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}} \quad \text{—} \ (1)$$

This is homogeneous differential eq in x and y to solve this put

$$y = vx$$

$$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$$

This eq (1) becomes

(7)

$$v + x\frac{dv}{dx} = \frac{\sqrt{x+vx} + \sqrt{x-vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$v + x \cdot \frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}} \times \frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}+\sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{1+\cancel{v}+1-\cancel{v}+2\sqrt{1-v^2}}{2v}$$

$$v + x\frac{dv}{dx} = \frac{2\,1+\sqrt{1-v^2}}{2v}$$

$$v + \frac{x\,dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

(8)

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2} - v^2}{v}$$

$$x\frac{dv}{dx} = \sqrt{1-v^2}\;\frac{(1+\sqrt{1-v^2})}{x}$$

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9

$$\frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \frac{dx}{x}$$

taking integral on b/s

$$\int \frac{v\,dv}{\sqrt{1-v^2}\left(1-\sqrt{1-v^2}\right)} = \int \frac{dx}{x}$$

put $1+\sqrt{1-v^2} = t$

$$\frac{1}{2}\left(1-v^2\right)^{-1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -\,dt$$

$$\int -\frac{dt}{t} = \int \frac{dx}{x}$$

10

- limit = ln + lnc

- $\ln(1+\sqrt{1-v^2} = \ln(cx$

$\ln(1+\sqrt{1-v^2} = -\ln(cx$

$\ln(1+\sqrt{1-v^2} = \ln((cx)^{-1}$

$1+\sqrt{1-v^2} = 1/cx$

$1+\sqrt{1-\frac{y^2}{x^2}} = 1/cx$

$1+\sqrt{\frac{x^2-y^2}{x^2}} = 1/cx$

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11

$$x + \sqrt{x^2 - y^2} = 1/c$$

$$x + \sqrt{x^2 - y^2} = c_1 \quad \therefore \quad 1/c = c_1$$

which is required

Q.NO (03)

12

$(D^4+D^2)y = 3x^2+4\sin x-2\cos x$

Sol:

$(D^4+D^2)y = 3x^2+4\sin x-2\cos x$

$\Rightarrow f(D)y = f(x)$

As it is non-homogenous linear equation so solution ~~wsb~~ will be

$y = y_c + y_p$ —(i)

complementry solution y_c

$D^4+D^2 = 0 \Rightarrow D^2(D^2+1) = 0$

Either $D^2 = 0 \Rightarrow D = 0$

$$D^2+1=0 \Rightarrow D^2=-1$$

$$D=\sqrt{-1} \Rightarrow D=i \text{ or } D=0+i$$

Roots are real and complex

$$y_c = C_1 e^{0x} + e^{0x}(C_2 \cos x + C_3 \sin x)$$

$$y_c = C_1 + C_2 \cos x + C_3 \sin x$$

$$y_p = \frac{1}{f(D)} F(x)$$

14

$$y_p = \frac{1}{D^4+D^2}(3x^2+4\sin x-2\cos x)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$f(D) = D^4 + D^2$

at $D=0 \Rightarrow f(D)=0$

So $f'(D) = 4D^3+2D$

Now also for $D=0 \Rightarrow f'(D)=0$
again differentiating

$f''(D) = 12D+2$

So for $D=0$

$f''(0) = 12(0)+2 = 2$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$\Rightarrow y_p = x^2 \frac{3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2} \cdot 2\cos x$

putting $D = 0$ in all

so

$$y_p = \frac{x^2 \, 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} - \frac{2x^2 \cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2 \cos x}{2}$$

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16

$$= \frac{3}{2}x^4 + 2x^2 \sin x - x^2 \cos x$$

So putting in equation (i)

~~$y = C_1 + C_2 \cos x + C_3 \cos$~~

$$y = C_1 + C_2 \cos x + C_3 \cos x + \tfrac{3}{2}x^4 + 2x^2 \sin x - x^2 \cos x$$

$$y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + \tfrac{3}{2}x^4$$