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Subject Probability & Statistic
Dept # BS(CS) 4th Semester
Assignment Final Term

Q1
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Ans: Solution:

$$S = \left\{ \begin{array}{l} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6) \\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6) \\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{array} \right\}$$

Let $A = \{$The sum is 7$\}$

$B = \{$The sum is Even$\}$

$C = \{$The sum is greater than 8$\}$

$D = \{$The two dice had the same outcome$\}$

So

$A = \{(1,6), (2,5), (3,4)\ (4,3)\ (5,2)\ (6,1)\}$

$B = \{(1,1), (1,3), (1,5), (2,2)\ (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6)\}$

$C = \{(3,6), (4,5), (4,6), (5,4), (5,5), (5,6), (6,3), (6,4), (6,5), (6,6)\}$

$D = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$A \cap B = \phi$

$A \cap C = \phi$

$A \cap D = \phi$

Now

$P(A) = 6/36 = 1/6$

$P(B) = 18/36 = 1/2$

$P(C) = 10/36 = 5/18$

$P(D) = 6/36 = 1/6$

$P(A \cap B) = 0$

$P(A \cap C) = 0$

$P(A \cap D) = 0$

Hence

$P(A/B) = \dfrac{P(A \cap B)}{P(B)} = 0/\tfrac{1}{2} = 0$

$P(A/C) = \dfrac{P(A \cap C)}{P(C)} = 0/\tfrac{5}{18} = 0$

$P(A/D) = \dfrac{P(A \cap D)}{P(D)} = 0/\tfrac{1}{6} = 0$

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Q2
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Ans
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When we are rolling two dice there are 36 different combination counting these up. There are 15 possibilities less than 7: (1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4) (3,1) (3,2) (3,3), (4,1), (5,1). The probability of getting less than 7, is

$$\frac{15}{36} = \frac{5}{12}$$

There are 6 possible combination of getting a 7: (1,6), (2,5) (3,4), (4,3) (5,2), (6,1) which gives a probability of

$$\frac{6}{36} = 1/6$$

This means that 21 possibilities account for getting less than or equal to 7 so there are 15 remaining possibilities of getting more than 7 so the probability must be 5/12 as well in calculating this we must assume that each combination is

Equally likely to fall as any other and therefore the dice are fair or else the calculation don't work.

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Q3 :

Ans : Solution :

Given That

$p = 2/3$, $n = 8$

$q = 1 - p$

$q = 1 - 2/3 = \frac{3-2}{3}$

$q = 1/3$

Let "x" denotes the number of games won by A then

(i) $P(x=4) = \binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4$

$= \frac{8!}{4!(8-4)!}\left(\frac{16}{81}\right)\left(\frac{1}{81}\right)$

$= \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{4! \, \cancel{4!}} \cdot \left(\frac{16}{6561}\right)$

$= \frac{8 \cdot 7 \cdot \cancel{6} \cdot 5}{\cancel{24}\,4} \quad \frac{\cancel{16}\,4}{6561}$

$\frac{1120}{6561} = \boxed{0.1707}$

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(ii) $P(x \geq 4) = 1 - P(x < 4)$

$$= 1 - \sum_{x=0}^{3} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= 1 - \left\{ \left(\frac{1}{3}\right)^{8} + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{7} + 28\left(\frac{2}{3}\right)^{2}\left(\frac{1}{3}\right)^{6} + 56\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5} \right\}$$

$$= 1 - \frac{1}{6561}\left\{1 + 16 + 112 + 448\right\}$$

$$= 1 - \frac{577}{6561}$$

$$= \frac{6561 - 577}{6561}$$

$$= \frac{5984}{6561}$$

$$= \boxed{0.9121}$$

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iii $$P(3 \le x \le 6) = \sum_{x=3}^{6} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3}\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5} + \binom{8}{4}\left(\frac{2}{3}\right)^{4}\left(\frac{1}{3}\right)^{4}$$
$$+ \binom{8}{5}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{3} + \binom{8}{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{2}$$

$$= \frac{8}{(3)^{8}}\left\{56 + 140 + 224 + 224\right\}$$

$$= \frac{8}{6561}\left\{56 + 140 + 224 + 224\right\}$$

$$= \frac{8 \times 644}{6561}$$

$$= \frac{5152}{6561}$$

$$= \boxed{0.7852}$$

Q4
5

Ans 5 Proof : Since the c_i's form a partition of the sample space we can apply the law of total probability for $A \cap B$.

$$P(A\cap B) = \sum_{i=1}^{M} P(A\cap B | c_i) P(c_i)$$

$$P(A\cap B) = \sum_{i=1}^{M} P(A|c_i) P(B|c_i) P(c_i)$$

(A and B are conditionally independent)

$$P(A\cap B) = \sum_{i=1}^{M} P(A|c_i) P(B) P(c_i)$$

= (B is independent of all c_i's)

$$P(A\cap B) = P(B) \sum_{i=1}^{M} P(A|c_i) P(c_i)$$

$$P(A\cap B) = P(B) P(A)$$

(law of total probability)

Hence A and B are independent.

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Q6
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Ans :- Binomial Distribution :-
5 Many experiment consist of repeated independent Trial each Trial having Two possible outcomes e.g The Two possible outcomes of a Trial my head and face success and failure. True and False etc.

The formula of binomial distribution is $P(x=n)\ f(x) = {}^{n}C_{x} P^{x} q^{n-x}$

Binomial frequency Distribution.

if the binomial probability distribution is multiplied by N then the number of experiment or sets the resulting distribution is known as the bi-nomial frequency distribution.

Formula :-

$$N \binom{n}{x} p^{x} q^{n-x}$$

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Q7
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Ans 3 coefficient of variation

For Data set A,

$$CV = \frac{\sigma}{\mu} * 100$$

$$CV = \frac{3}{45} \times 100$$

$$\boxed{CV = 6.7}$$

For Data set B,

$$CV = \frac{11}{60} \times 100$$

$$\boxed{CV = 18.3}$$

For Data set C,

$$CV = 5/50 \times 100$$

$$\boxed{CV = 10}$$

For Data set D,

$$CV = 15/25 \times 100$$

$$\boxed{CV = 60}$$

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Q5
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Ans 5 The probability function for a binomial random variable is

$$b(x; n, p) = \binom{n}{x} p^x q^{n-x}$$

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x} \qquad = q = 1-p$$

This is probability of having x success in a series of n independent Trials is p if x is random variable

with this probability distribution

$$E(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

Since $x=0$ Term vanishes let $y = x-1$ and $m = n-1$ subbing $x = y-1$ and $n = m+1$ into the last

$$E(x) = \sum_{y=0}^{m} \frac{(m+1)!}{y!(m-y)!} p^{y+1} (1-p)^{m-y}$$

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$$= (m+1) p \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^{y} (1-p)^{m-y}$$

$$= np \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^{y} (1-p)^{m-y}$$

By Bionomial Theorem

$$(a+b)^{m} = \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} a^{y} b^{m-y}$$

set $a = p$ and $b = 1-p$

$$= \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^{y} (1-p)^{m-y}$$

$$= \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} a^{y} b^{m-y}$$

$$= (a+b)^{m}$$

$$= (\cancel{p}+1-\cancel{p})^{m}$$

$$= 1$$

So Read

$$\boxed{E(X) = np}$$

Similarly but this time using $y = x-2$ and $m = n-2$

$$E(x(x-1)) = \sum_{x=0}^{n} x(x-1)\binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=2}^{n} \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{y=0}^{m} \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y}$$

$$= n(n-1)p^2 (p+(1-p))^m$$

$$= n(n-1)p^2$$

So the variable of x is

$$E(x^2) - E(x)^2 = E(x(x-1)) + E(X) -$$

$$E(x)^2 = n(n-1)p^2 + np - (np)(np)$$

$$\boxed{= np(1-p)}$$