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Subject : Numerical Analysis

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to

Semester : 11th

Day: M T W T F S S

Question no 1

Q Find the root of the equation

$$x^3 + 3.993 \times 10^{-4} = 0.165x^2$$

Use Newton raphson method with

$x_0 = 0.02$

Sol: Rearranging equation

$$x^3 - 0.165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - f(x_n)/f'(x_n)$$

$$f'(x) = 3x^2 - 0.33x = 0$$

→ $x_1 = x_0 - f(x_0)/f'(x_0)$

$= 0.02 - \frac{0.00033}{-0.0054}$

$\boxed{x_1 = 0.081}$

$f(x_0) = 0.00033$

$f'(x_0) = -0.0054$

→ $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.081 - \frac{(-0.00016)}{-0.0070}$

$\boxed{x_2 = 0.058}$

$f(x_1) = -0.00016$

$f'(x_1) = -0.0070$

→ $x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.058 - \frac{0.000030}{(-0.0090)}$

$\boxed{x_3 = 0.061}$

$f(x_2) = 0.000030$

$f'(x_2) = -0.0090$

Question no 2

Q Use the numbers to find the lagrange interpolation Polynomial for $f(x)$ at $x=3$

Ans

$x_0 = 2$, $y_0 = 0.5$

$x_1 = 2.75$, $y_1 = 0.36$

$x_2 = 4$, $y_2 = 0.25$

As we know that

Lagrange Interpolation formula

$$y = (x-x_1)(x-x_2)\cdots(x-x_n) / (x_0-x_1)(x_0-x_2)\cdots(x_0-x_n) \cdot y_0$$

$x_0 = 2$, $y_0 = 0.5$

$x_1 = 2.75$, $y_1 = 0.36$

$x_2 = 4$, $y_2 = 0.25$

$$y = \frac{(x-(x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x-x_2} y_1 +$$

$$\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x-x_1)} y_2$$

Putting value

$$y = \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)}(0.5) + \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)}(0.36)$$

$$+ \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)}(0.25)$$

$$y = (-0.833) + 0.384 + 0.025$$

$$\boxed{y = -0.424}$$

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Question no 3

Q3 Complete the divided difference table for the given and construct the interpolating polynomial that uses all this data.

$x = 1.0 \quad 1.3 \quad 1.6 \quad 1.9 \quad 2.2$

$y = 0.7651977 \quad 0.6200860 \quad 0.4554022 \quad 0.2818186 \quad 0.1103623$

x_i	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i-1}, x_i)$	$f(x_i - x \cdots, x_1)$	$f(x_{i+1} - x_i)$
x_0 1	0.7651977				
		-0.4837056			
x_1 1.3	0.6200860		-0.108734		
		-0.548946		0.0658785	
x_2 1.6	0.4554022		-0.0494433		-0.0028819
		-0.578612		0.0625255	
x_3 1.9	0.2818186		0.086818		
		-0.571521			
x_4 2.2	0.1103623				

i) $$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \frac{0.6200860 - 0.7651977}{1.3 - 1}$$

$$\boxed{f(x_0, x_1) = -0.4837056}$$

ii) $$f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{-0.4554022 - 0.6200860}{1.6 - 1.3}$$

$$\boxed{f(x_1, x_2) = 0.548946}$$

3) $$f(x_2, x_3) = \frac{f(x_3) - f(x_2)}{x_3 - x_2}$$

$$= \frac{0.2818186 - 0.4554022}{1.9 - 1.6}$$

$$\boxed{f(x_2, x_3) = -0.5778612}$$

4) $$f(x_3, x_4) = \frac{f(x_4) - f(x_3)}{x_4 - x_3}$$

$$= \frac{0.1103623 - 0.2818186}{2.2 - 1.9}$$

$$\boxed{f(x_3, x_4) = 0.571521}$$

Second divided difference

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$= \frac{-0.548946 - (-0.4837056)}{1.6 - 1}$$

$$= -0.108734$$

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{-0.578612 - (-0.548946)}{1.9 - 1.3}$$

$$f(x_1, x_2, x_3) = -0.0494433$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{-0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$= 0.006818$$

Third divided difference.

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{0.0494433 - (-0.108734)}{1.9 - 1}$$

$$= 0.0658785$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.0506818 - (-0.049443)}{2.2 - 1.3}$$

$$= 0.0625125$$

Fourth divided difference:

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{0.0625125 - 0.0658785}{2.2 - 1}$$

$$= 0.002804$$

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$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3) + (x - x_0)(x - x_1)(x - x_2)(x - x_3) f(x_0, x_1, x_2, x_3, x_4)$$

$$= 0.7651977 + (x - 1)(-0.4837056) + (x - 1)(x - 1.3)(-0.68734) + (x - 1)(x - 1.3)(x - 1.6)(0.0658785) + (x - 1)(x - 1.3)(x - 1.6)(x - 1.9)(0.0018237)$$