I.D = 7985

Section = B

Department = Civil

PaPer = Differential equation

Submitted to

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ii) No of non zero ↗ in a Echlen form is called rank

iii) a = 8

iv) = 3

v) Scalar Matrix

vi) $\log y = x - x^2 + c$ or $y = e^{x - x^2} + c$

vii) Order 1 degree = 3

viii) Order and degree is define as its not polynomial

ix) $Y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{61} + 5 \rightarrow$ homogenous equation

x) $bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b$

oR

$b^2c(c-1) + ab(b-a) + ac(a-c)$

Express the Determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product factor in which are the linear in a, b. c

Sol

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{bmatrix} a^2 & c^2 \\ a^3 & c^3 \end{bmatrix} + c\begin{bmatrix} a^2 & b^2 \\ a^3 & b^3 \end{bmatrix}$$

$$= a(a^2b^2 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$+$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3b^2 + a^2b^3 - a^3b^2c$$

common abc

$$\Rightarrow abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$$

$$\Rightarrow \boxed{abc(bc(c-b) - ac(c+a) + ab(b-a))}$$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Find the Eigen Value

Sol

Characteristic equ $[A-\lambda I]=0 \rightarrow A$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinet

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} -(-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0$$

Again

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expand by R_1

$$\Rightarrow 3-\lambda \begin{bmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} -(-1) \begin{bmatrix} -1 & -1 \\ -1 & 2-\lambda \end{bmatrix} -1 \begin{bmatrix} -1 & 3-\lambda \\ -1 & -1 \end{bmatrix}$$

$$= (3-\lambda)\Big[(3-\lambda)(2-\lambda)-(-1)(-1)+1((-1)(2-\lambda)$$

$= (3-\lambda)(\lambda^2 - 5\lambda + 5) + (-3+\lambda) - (4-\lambda)$

$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$

$= -\cancel{\lambda^3 + 15}$

$= \boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \rightarrow (a)$

$$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$\Rightarrow -1(6 - 3\lambda - 2\lambda + \lambda^2 - 1) + (-2 + \lambda - 1)$

$\Rightarrow -\lambda^2 + 5\lambda - 5 - 3 + \lambda$

$= -\lambda^2 + 5\lambda - 5 - 3 + \lambda$

equ (B)

$$\Rightarrow -1\begin{bmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix}$$

Expand by C_1

$$-\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right)$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \rightarrow (c)$$

put a, b and c in equ (B)

$$= (2-\lambda)[-\lambda^3+8\lambda^2-18\lambda+8]-\lambda^2+6\lambda-8-\lambda^2$$
$$+6-8$$

$$= -2\lambda^2+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2$$
$$+6\lambda-8-\lambda^2+16\lambda-8$$

$-8\lambda + 6\lambda + 6\lambda + 16 - 16$

$\Rightarrow \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0$

By a synthatic division we get

$\lambda(\lambda-2)(\lambda^2 - 8\lambda + 16) = 0$

$\lambda = 0$

$\lambda - 2 = 0 \Rightarrow \boxed{\lambda = 2}$

$\lambda^2 - 8\lambda + 16 = 0$

By factorization method

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda-4) - 4(\lambda-4) = 0$

$(\lambda-4)(\lambda-4)$

$\lambda = 4, \ \lambda = 4$

$\boxed{\lambda_1 = 0, \ \lambda_2 = 2, \ \lambda_3 = 4, \ \lambda_4 = 4}$

Q 3

$$(x^2 + 3y^2)\,dx - 2xy\,dy = 0$$

$x = 2$ and $y = 6$

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

This is the hemogenous differential equation

To Solve it put $y = Vx$

Differentiate w.r.t x

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

So the given differential equation becomes

$$v + x\frac{dv}{dx} = \frac{x^2 + 3(vx)^2}{2x(vx)}$$

$$V + x\frac{dv}{dx} = \frac{x^2 + 3v^2x^2}{2x^2 v}$$

$$V + x\frac{dv}{dx} = \frac{x^2(1 + 3v^2)}{2x^2 v} = \frac{3v^2 + 1}{2v}$$

$$\frac{x\,dv}{dx} = \frac{3v^2+1}{2v} - v$$

$$= \frac{3v^2+1-2v}{2v}$$

$$= \frac{v^2+1}{2v}$$

$$\frac{2v}{v^2+1}\,dv = \frac{1}{x}\,dx$$

due to integration

$$\int \frac{2v}{v^2+1}\,dv = \int \frac{1}{x}\,dx$$

$$\ln(v^2+1) = \ln x + \ln c$$

$$\cancel{\ln}(v^2+1) = \cancel{\ln}(xc)$$

$$v^2+1 = xc$$

As $y = vx$, so $v = y/x$

$$\left(\frac{y}{x}\right)^2 + 1 = xc$$

$$\frac{y^2}{x^2} + 1 = xc$$

$$\frac{y^2+x^2}{x^2} = xc \Rightarrow y^2+x^2=x^3c \to C$$

So put x and y value

in equ (c)

$$(6)^2 + (2)^2 = (2)^3 c$$

$$36 + 4 = 8c$$

$$\frac{\overset{5}{\cancel{40}}}{\cancel{8}} = \frac{8c}{\cancel{8}}$$

$$5 = c$$

$$\boxed{c = 5}$$ equ (D)

So put equ (D) in (C)

$$y^2 + x^2 = 5x^3$$

$$y^2 = x^2 - 5x^3$$

$$y^2 = x^2(x-1)$$

Taking squar root on both sides

$$y = \sqrt{x^2(x-1)}$$

$$\boxed{y = \pm x^2\sqrt{(x-1)}} \rightarrow \text{Ans}$$