ID	7655
Subject	Applied calculus
Submitted to	Maadam Shumaila
Submitted by	Syed USAMA
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Q1 The function $g(t)$ is defined by

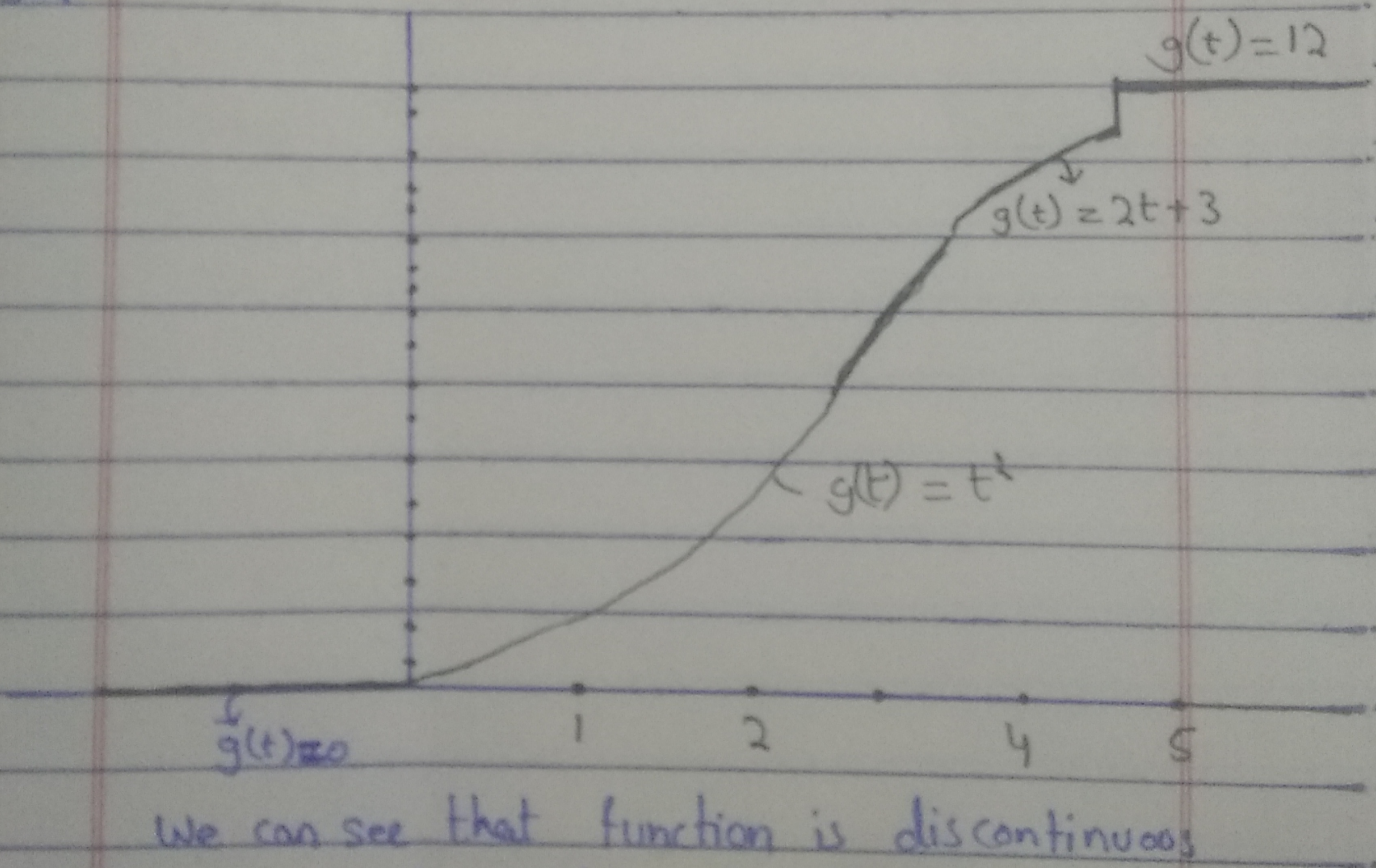
$$g(t) = 0 \qquad t < 0$$

$$t^2 \qquad 0 \leq t \leq 3$$

$$2t + 3, \quad 3 < t < 4$$

a) State any point of discontinuity;

Sol



We can see that function is discontinuous at point $t = 4$

$\Rightarrow$ because $\lim\limits_{t \to 4} g = 12$ $\longrightarrow$ $\lim\limits_{t=4^-} g \neq \lim\limits_{t=4^+} g$

$\Rightarrow$ $\lim\limits_{t \to 4} g = 2t+3 = 11$ hence g is discontinuous at 4.

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Sol)

$$g(t) = \int_0^3 t^2\,dt$$

$$= \frac{t^3}{2t} = \frac{t^2}{2}\Big|_0^3 = \frac{9}{2} - 0$$

$$= \frac{9}{2}$$

$$g(t) = \int_3^4 2t+3\,dt$$

$$= \frac{2t^2+3t}{2}\Big|_3^4 = \frac{32+12}{2} - \frac{18+9}{2}$$

$$= \frac{44}{2} - \frac{37}{2} = 22-18.5$$

$$= 3.5$$

$$g(t) = \int_4^\alpha 12\,dt = 12t\Big|_4^\alpha = \alpha$$

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b Find if the exist

i) $\lim_{t \to 3} g$.

Sol Now

$$\lim_{t \to 3} g = \lim_{t \to 3} t^2 = 3^2 = 9$$

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Q2 $y(x) = x^2 + \sin x$

Sol

$y(0) = (0)^2 + \sin(0)$

$= 0 + 0 = 0$

$y' = \frac{d}{dx}(x)^2 + \sin(x)$

$y'(x) = 2x + \cos x$

$y'(0) = 2(0) + \cos(0)$

$y'(0) = 0 + 1 = 1$

$y''(x) = 2 - \sin x$

$y''(x) = 2 - \sin(x)$

$y''(0) = 2 - \sin(0) = 2 - 0 = 2$

$y'''(x) = 0 - \cos x$

$= -\cos x$

$y'''(0) = -\cos(0)$

$= -1$

Now

Using Maclaring Series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3 f'''(0)}{3!} + \ldots\ldots + \frac{x^{n-1}}{(n-1)!} f^{n-1}(0)$$

$$y(x) = y(0) + x\, y(0) + \frac{x^2}{2!} y''(0) + \frac{x^3}{3!} y'''(0) + \ldots\ldots$$

$$x^2 + \sin x = 0 + x(1) + \frac{x^2}{2}(2) + \frac{x^3}{6}(-1) + \ldots\ldots$$

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$x^2 + \sin x = x + \cancel{x^2} - \frac{x^3}{6} + \ldots\ldots$

Q3

i) $1 + xy = x^2 + y^2$

$y'' = ?$

$y' = 1 + xy - x^2 - y^2$

$\frac{d}{dx}(1 + xy) = \frac{d}{dx}(x^2 + y^2)$

$0 + x\frac{dy}{dx} + y\frac{dx}{dx} = 2x + 2y\frac{dy}{dx}$

$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$

$\frac{dy}{dx}(x - 2y) = 2x - y$

$dy/dx = \frac{2x - y}{x - 2y}$

$\frac{d^2y}{dx^2} = ?$

Now

$\frac{d^2y}{dx^2} = \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$

$y'' = \frac{(x-2y)(2 - dy/dx) - (2x-y)(1 - 2\,dy/dx)}{(x-2y)^2}$

$$y'' = \frac{2x - x\frac{dy}{dx} - 4y + 2y\frac{dy}{dx} - 2x + 4x\frac{dy}{dx} \cancel{- 2y\frac{dy}{dx}} + y}{(x-2y)^2}$$

$$y'' = \frac{-x\frac{2y}{dx} - 4y + 4x\frac{dy}{dx} + y}{(x-2y)^2}$$

$$y'' = \frac{-x\frac{dy}{dx} - 4y + 4x\frac{dy}{dx} + y}{(x-2y)^2}$$

$$y'' = \frac{\frac{dy}{dx}(3x) - 3y}{(x-2y)^2}$$

$$y'' = \frac{\frac{2x-y}{x-2y}(3x) - 3y}{x-2y}$$

$$y'' = \frac{6x-3xy}{(x-2y)} \times \frac{1}{x-2y} - \frac{3y}{x-2y}$$

$$\frac{d^2y}{dx^2} = \frac{6x-3xy}{(x-2y)^2} - \frac{3y}{(x-2y)}$$

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Q3 b) Find logrithmic derivation ;

$$y = x^3 (1+x)^9 e^{6x}$$

Sol

$$y = x^3 (1+x)^9 e^{6x}$$

Now by applying logrithmic differentiation

we use ;

$$y' = \ln\left[x^3 (1+x)^9 e^{6x}\right]$$

Now

$$y' = \ln x^3 + \ln(1+x)^9 + \ln(e^{6x})$$

$$y' = \frac{1}{x^3} + 9\ln(1+x) + 6x$$

$$y' = \frac{1}{x^3} + \frac{9}{1+x} + 6x$$

Ans