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Course: Signal and Systems

Q1(a) :=>

let $x(t)$ be continuse time signal
Then

$$x(t) = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} x(J\omega) e^{J\omega t} d\omega$$

Diff w.r.t t

$$\frac{d}{dt} x(t) = \frac{1}{2\pi} \int_{-x}^{x} x(J\omega) d\omega \frac{d}{dt} e^{J\omega t}$$

$$\frac{dx(t)}{dt} = \frac{1}{2x} \left[\int_{-x}^{x} x(j\omega) \{ e^{j\omega t} j\omega) d\omega \right.$$

$$\frac{dx(t)}{dt} = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} [j\omega x(j\omega)] e^{j\omega t} d\omega$$

$$F\left\{\frac{d}{dt} x(t)\right\} = j\omega \; X(j\omega)$$

Result: We conclude that if a fle is differantial in time domain It is multiplied ~~by~~ $j\omega$ in frequency domain.

$\longrightarrow X \Longrightarrow X \Rightarrow$

Q1 (B): $x[n] = 2\delta[n] - 4$
$h[n] = 3\delta[n] + 6$

Solution: $Y(Z) = H(Z)\,X(Z)$.
find $Y[n]$
$X(Z) = 2 - 4Z^{-2} + 2Z^{-3}$
$H(Z) = 3 + 1Z^{-1} + 2Z^{-2}$
Now:-
$Y(Z) = H(Z)\;X(Z)$
$= (2 - 4z^{-2}) + 2z^{-3})(3 + (z^{-1} + 2z^{-2})$
$= 6 + 2z^{-1} + 4z^{-2} - 12z^{-2} - 4z^{-3} - 8z^{-4}$
$+ 6z^{-3} + 2z^{-4} + 4z^{-5}$
$= 6 + 2z^{-1} - 8z^{-2}\; 2z^{-3} + 6z^{-4} + 4z^{-5}$

To find $Y(n)$ use time delay property

$\{ Y[n] = 6\delta[n] + 2\delta(n-1) - 8\delta(n-2)\}$
$\{ -2\delta[n-3] + 6\delta[n-4] + 4\delta[n-5]\}$.

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Q2: $f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$

Solution: $a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx.$

$= 1/2\pi \left[\int_{-\pi}^{0} f(x)\, dx + \int_{0}^{\pi} f(x) dx \right]$

$= 1/2\pi \int_{-\pi}^{0} -\pi/2\, dx + \int_{0}^{\pi} \pi/2\, dx.$

$= \frac{1}{2}\pi \left[\frac{-\pi}{2} \int_{-\pi}^{0} 1 dx + \int_{0}^{\pi} 1 dx \right]$

$= 1/2\pi \left[-\pi/_x\, x \Big|_{-\pi}^{0} + \frac{\pi}{2} x \Big|_{0}^{\pi} \right]$

$= 1/2\pi \left[\frac{\pi}{2}(0-(-\pi)) + \pi/2 [\pi - 0] \right]$

$= 1/2\pi \left[-\frac{\pi}{2}(\pi) + \frac{\pi}{2}(\pi) \right]$

$= 1/2\pi \left[-\frac{\pi^2}{2}(\pi) + \frac{\pi}{2}(\pi) \right]$

$= 1/2\pi \left[-\frac{\pi^2}{2} + \pi^2/2 \right].$

$= 1/2\pi [0/2] \Rightarrow (a = 0)$

NOW:-

$a_x = 1/\pi \int_{-\pi}^{\pi} f(x) \cos nx dx$

$= 1/\pi \int_{-\pi}^{0} f(x) \cos nx dx + \int_{0}^{\pi} f(x) \cos nx dx.$

$= 1/\pi \int_{-\pi}^{0} \pi/2 \cos nx dx + \int_{0}^{\pi} \pi/2 \cos nx dx.$

$= 1/\pi \left[\pi/2 \frac{\sin nx}{n} \Big|_{\pi}^{0} + \frac{\pi}{2} \frac{\sin nx}{n} \Big|_{0}^{\pi} \right]$

$= \frac{1}{n\pi} \left[\frac{-\pi}{2} \sin n(0) - \sin n(-\pi) + \frac{\pi}{2} [\sin n(\pi) - \sin n(0)] \right]$

$$= \frac{1}{n\pi}\left(\frac{-\pi}{2}(0) + \pi/2 (0)\right]$$

$$= 1/n\pi (0)$$

Now:

$$b_n = 1/\pi \int_{\pi}^{\pi} \sin x\,dx.$$

$$= \frac{1}{\pi}\left[\int_{-x}^{0} f(x) \sin x\,dx + \int_{0}^{\pi} f(x) \sin x\,dx\right.$$

$$= \frac{1}{\pi}\left(\int_{\pi}^{0} \sin x\,dx + \int \frac{\pi}{2} \sin x\,dx\right].$$

$$= 1/\pi\left(-\pi/2 \int_{-x}^{0} \sin x\,dx + \frac{\pi}{2}\int_{0}^{\pi} \sin nx\,dx\right.$$

$$= 1/\pi\left[\frac{-\pi}{2} - \frac{\cos nx}{n}\Big|_{-\pi}^{0} + \pi/2 \frac{-\cos nx}{n}\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}\left(-\cos n(0) + \cos x(-\pi)\right)\right] + \frac{\pi}{2}\left[\frac{-\cos n(\pi) +}{\cos n(0)}\right]$$

$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}\left[-1 + \cos n(-\pi)\right]\right) + \frac{\pi}{2}\left(-\cos\pi + \cos n(0)\right)\Big]$$

$$\frac{\pi}{2} = \frac{1}{n\pi}\left[-1\left[-1 \cos n(-\pi)\right]\right] + 1\left[-\cos nx + 1\right]\Big]$$

$$= \frac{1}{2n}\left(1 - \cos n\pi - \cos n\pi + 1\right]$$

$$- \frac{1}{2n}\left[2 - 2\cos n\pi\right]$$

Now

$$b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ 4/2n & \text{if } n \text{ is odd.}\end{cases}$$

$$\left\{ b_n = 4/2n \right\}$$

$$f(x) = a_0 + a_1\cos x + a_2\cos 2x + a_3\cos 3x + \cdots$$
$$b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \cdots$$

~~f(x) = ...~~

PTO.

(5)

$f(x) = (0) + (0)\cos x + 0\cos(2x) + 0\cos(3x)$
$+ \frac{4}{2}\sin x + (0)\sin 2x + \frac{4}{3(2)}\sin 3x + \cdots$

$f(x) = \frac{4}{2}\sin x + \frac{4}{6}\sin 3x + \cdots)$ Ans.

=X=X=X=

Q3:

Ans: $X(Z) = \frac{2Z^2+2Z}{Z^2+2Z-3}$

$X(Z) = \frac{2Z(Z+1)}{Z^2+3Z-Z-3}$

$= 2Z(Z+1)/(Z+3)(Z-1)$

or

$\frac{2(Z+1)}{Z^2+2Z+3} = \frac{A}{(Z+3)} + \frac{B}{(Z-1)} \longrightarrow (1)$

put $Z=1$ & multiplying both side by $(Z+3)(Z-1)$

put $Z=1$

This eq (1) become

$2(Z+1) = A(Z-1) + B(Z+3) - (ii)$

put $Z=1$ in (ii)

$2(1+1) = B(1+3)$
$4 = 4B$
$(B=1)$

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put $Z = -3$ in eq → (ii)

$2(-3+1) = A(-3-1)$

$-4 = -4A$

$A = 1$

Now put (A) and (B) in eq (ii)

$$\frac{2(Z+1)}{(Z+3)(Z+1)} = \frac{1}{(Z+3)} + \frac{1}{(Z-1)}$$

So Inverse Z-Transform

$x(Z^{-1}) = u[k] + 1(-1)^k$

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Q4:-

Ans:- As we are given

$$A = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = [1, 2] \; D = [0]$$

$G(s) = C[SI - A]^{-1} B + D$

$G(s) = C[SI - A]^{-1} B + D$

$$= [1 \;\; 2]\left[S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}\right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= [1 \;\; 2] \frac{1}{s(s+2)+1} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= [1 \;\; 2] \frac{1}{s^2+2s+1} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+2s+1} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$= G(s) = \frac{1}{s^2+2s+1} [s-2]$$

[num, den] = ss2tf(A,B,C,D)

[A,B,C,D) = tf2ss [num, den)

$= X = X = X = X = X =$

Q5:-

Sol: The fourier Transform of the Given ftn $x(t)$ is Giving by :-

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt.$$

$$\because\ e^{-a|t|} = \begin{cases} e^{ax} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0 \end{cases}$$

$$X(j\omega) = \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{e^{(a-j\omega)t}}{a-j\omega}\Bigg|_{-\infty}^{0} + \frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\Bigg|_{0}^{\infty}$$

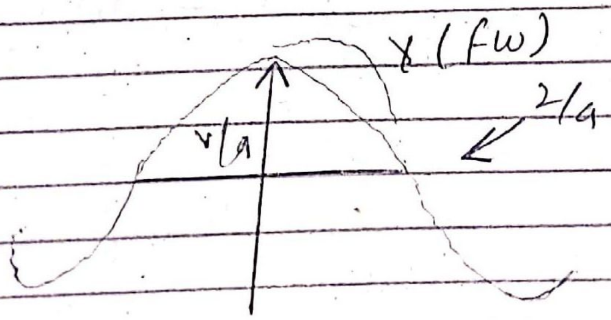
$$= \frac{1}{(a-j\omega)}\left[e^{0} - e^{-\infty}\right] - \frac{1}{a+j\omega}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{1}{(a-jw)}(1-0) - \frac{1}{a+jw}(0-1)$$

$$= \frac{1}{a-jw} + \frac{1}{a+jw}$$

$$= \frac{a+jw+a-jw}{a^2-(jw)^2}$$

$$= x(jw) = \frac{2a}{a^2+w^2}$$



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