Department of Electrical Engineering
Assignment
Date: 13/04/2020

Course Details

Course Title: ___ Digital Signal Processing ______ **Module:** _ 6th ______
Instructor: ___ **Total Marks:** _ 30 ______

Student Details

Name: **ABDUL BASIT** **Student ID:** **13684**

Q1.	(a)	Consider the following analog signal $$x_a(t) = 3cos100\pi t + 4sin200\pi t$$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$ This is signal is sampled at the rate $F_s = 2Hz$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $$x[n] = \{ 2, \underset{\uparrow}{1}, -2, 3, -4\} \quad , h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$	Marks 5 CLO 2

	(b)	Compute the convolution y(n) of the following signal $$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$ $$h(n) = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $$x(n) = \begin{cases} \left(\frac{1}{4}\right)^{n}, & n \ge 0 \\ \left(\frac{1}{3}\right)^{-n}, & n < 0 \end{cases}$$ ii. $$x(n) = \begin{cases} \left(\frac{1}{2}\right)^{n} - 3^{n}, & n \ge 0 \\ 0, & elsewhere \end{cases}$$	Marks 10 CLO 2

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Q1-

(a) Consider the following analog signal.

$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$.

Solution:

(1) Minimum Sampling rate.

$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

$f_1 = 50Hz$, $f_2 = 100$ $f_3 = 100Hz$

So

f_3 is max (greater than f_1).

$f_1 = 50Hz$ is minimum sampling rate

to avoid aliasing.

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(2)

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As from equation

$$x_a(t) = 3\cos \frac{100\pi t}{100} + 4\sin \frac{200\pi t}{100}$$

$$= 3\cos \pi t + 4\sin 2\pi t.$$

So
the effect of this sampling rate on the newly generated discrete time is that there will be no Aliasing phenomenon mean there will be present components in the reconstru of the signal and we can recon the orignal signal.

Orignal signal.

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(3)

(c)

Hence for ideal interpolation we can construct the orignal signal. & also frequency component, $f_1 = 50Hz$, $f_2 = 100Hz$

$$y_a(t) = 3\cos 100\pi t + 4\sin 200\pi t.$$

The orignal signal is different from it which will constroted because we use sampling frequency. - This distortion of the orignal analog signal was caused by the aliasing effect

Q1 (b).

Consider a discrete time signal which is given by

$$x(n) = \begin{cases} 0.5^n, & n > 0 \\ 0, & n < 0 \end{cases}$$

This is signal is sampled in the rate $F_s = 2Hz$.

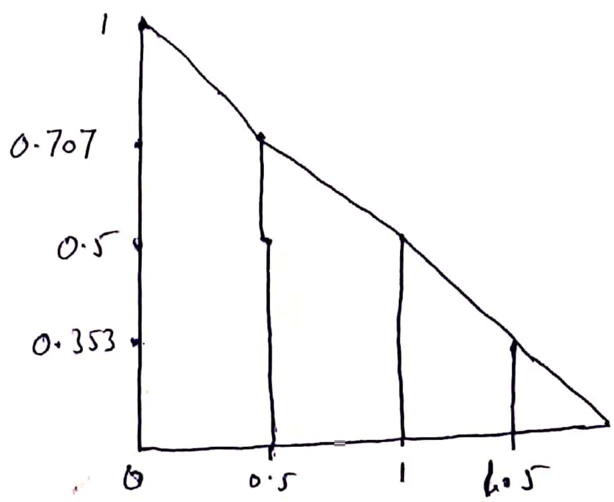
Solution

$F_s = 2Hz$

$$F_s = \frac{1}{T} = T = \frac{1}{F_s} = \frac{1}{2} = 0.5 \text{ sec.}$$

(1) Draw the sampled signal.

$x(n)$	$= 0.5^n$
0	1
0.5	0.7071
1	0.5
1.5	0.353



(5)

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(7)

Quantization level

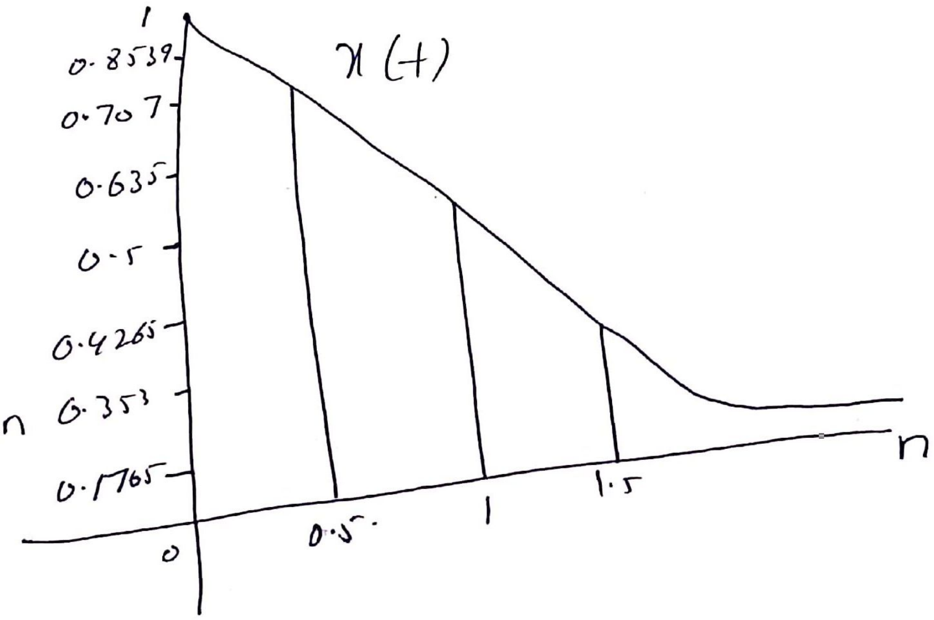
$l = 2^n$

$n = \text{bits} = 3.$

$$\text{Resolution} = \frac{X_{max} - Y_{min}}{L}$$

$$= \frac{1-0}{8} = 0.0125$$

Rang of Quantization



(6)

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iii Tabular form.

n	$x(n)$	$x_q(n)$ Trucation	$x_q(n)$ Rounding	$x_q(n) - x(n)$
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	0.0465
2	0.707	0.7	0.7	-0.07
3	0.6035	0.6	0.6	-0.035
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	-0.0265
6	0.353	0.3	0.4	0.047
7	0.1765	0.1	0.2	0.0235

Q2 (a)

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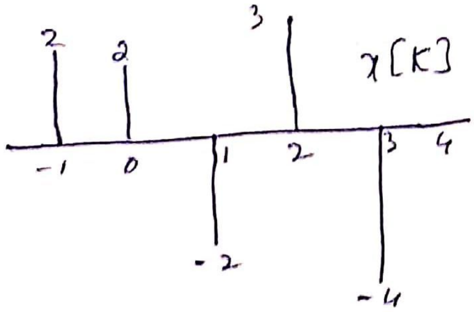
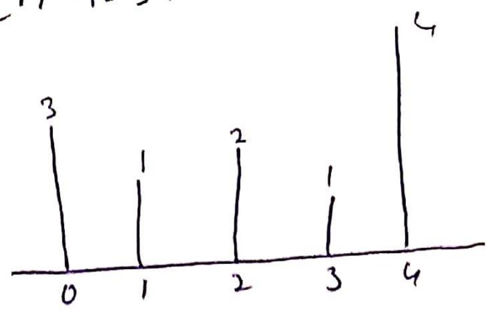
1. Determine the response of the system of the following input signal and given impulse respone

$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\} \quad h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

<u>Sol</u>

$$y[n] = \sum_{K=\infty}^{\infty} x[K]\, h[n-K].$$





h(-K) folded signal

$$y[0] = \sum_{K=-1}^{0} x[-1]h(-1) + x(0)\, h(0)$$

$$= 2\times 1 + (1)(3)$$

$$= 2+3$$

$$= 5$$

(8)

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For $n=1$

$h(1-K)$

$$y[1] = \sum_{K=-1}^{1} x[n]\, h[1-K].$$

$= x(-1)\,K(-1) + x(0)\,h(0) + x(1)\,h(1)$

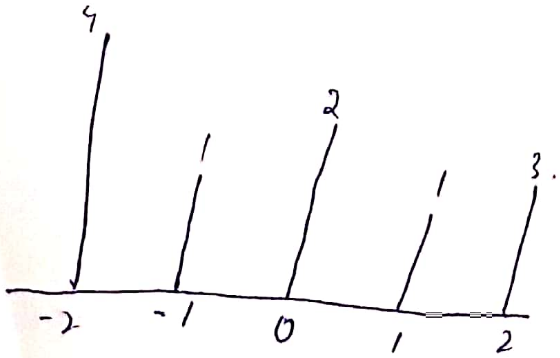
$= (2)(2) + (1)(1) + (3)(-2)$

$= 4 + 1 - 6$

$= -1$

$n = 2$

$h[2-K]$



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$$y[2] = \sum_{k=-1}^{2} x[n]\, h[2-k]$$

$$= x(-1)\, h(-1) + x(0) + h(0) + x(1)\, h(1)$$
$$+ x(2)\, h(2)$$

$$= (2)(1) + (1)(2) + (-2)(1) + (3)(3)$$
$$= 2+2-2+9$$
$$= 11$$

$n = 3$

$$y[3] = \sum_{k=-1}^{3} x(n)\, h(3-k)$$

$$= x(-1)\, h(-1) + x(0)\, h(0) + x(1) h(1) + x(2)$$
$$+ x(3)\, h(3)$$

$$= 2\times4 + (1)(1) + (-2)(2) + (3)(1) + (-4)$$
$$= 4+1-4+3-12$$
$$= -8.$$

10

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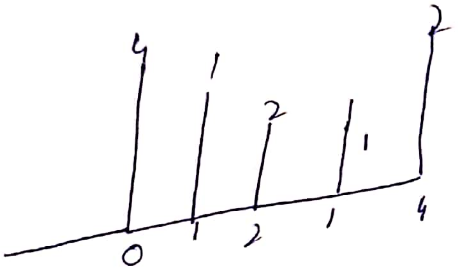
$n = 4$



$h[4-K]$.

$$y[4] = \sum_{K=0}^{3} x(n)\, h[4-K)$$

$$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

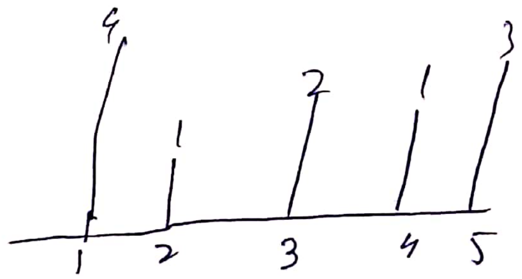
$$= (1\times 4) + (-2)(1) + (3)(2) + (-4)(1)$$

$$= 4 - 2 + 6 - 4$$

$y(4] = 4$.

$n = 5$

$h[5-K]$.



11

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$$y[5] = \sum_{k=1}^{3} x(k)\,h(5-k)$$

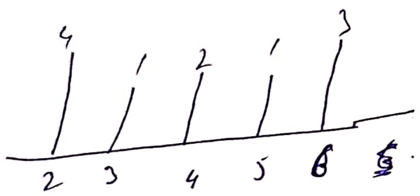
$$= x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= (-2)(4) + (3)(1) + (-4)(2)$$

$$= -8+3-8$$

$$= -13.$$

$n = -6$



$$g(6) = \sum_{k=2}^{k=} x(2)h(2) + x(3)h(3)$$

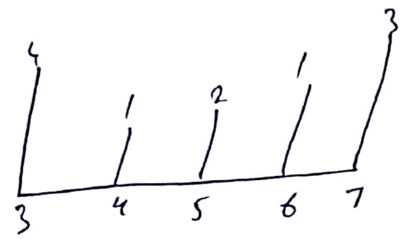
$$y[6] = (3)(4) + (1)(-4)$$

$$= 12-4$$

$$= 8.$$

(12)

$n = 7$



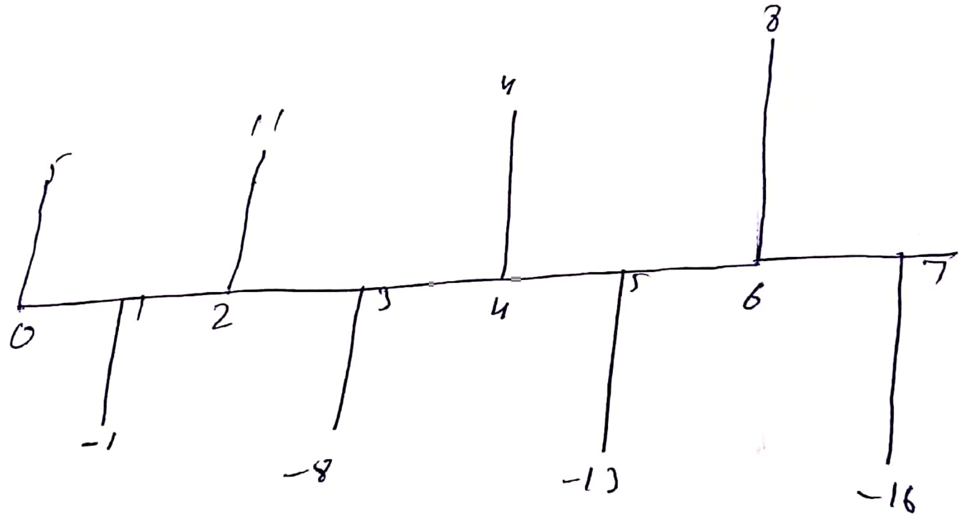
$h(7-K)$.

$y[7] = x(3)h(3)$

$= 4 \times (-4$

$= -16.$

$y[n] =$



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Q2 (b) Compute the convolution $y[n]$ (13) of the following signal

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$$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & \text{elsewhere.} \end{cases}$$

Sol

$x(n) = \{\alpha^{-2}, \alpha^{-1}, \alpha, \alpha^{1}, \alpha^{2}, \alpha^{3}, \alpha^{4}, \alpha^{5}, \alpha^{6}\}.$

and

$h(n) = \{1, 2, 4, 8, 16\}.$

$$y(n) = \sum_{k=0}^{4} h(k)x(n-k)$$

There fore

$y(-2) = \alpha^{-2}$

$y(-1) = x(-\alpha) + x(1) = \alpha^{-2} + \alpha^{-1}.$

$y(0) = \cancel{h}(x)x(-2) + h(x)\cancel{x}(-9) + h(x)\, x(\alpha)$

$= 9 - \alpha^{-2} + 9 \cdot \alpha^{-2} + 9 \cdot 9 + \alpha^{-3} + 9.$

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(14) Abdul-Basit 13684

$y(1) = \alpha^{-2} + \alpha^{-3} + 9 + h(1)\, x(4-3)$

$= \alpha^{-2} + \alpha^{-1}\, 9 + h(3)(x(1)) = x^{-2} + x^{-1} + 1 + 2\alpha^{1}$

$y(2) = \alpha^{-2} + \alpha^{-1} + 1 + 2\alpha^{1} + h(2)x(2)$

$= \alpha^{-2} + \alpha^{-3} + 1 + 2\alpha^{1} + 4\alpha^{2}.$

$y(3) = \alpha^{-2} + \alpha^{-1} + 9 + 2\alpha^{1} + 4\alpha^{2} + 8\alpha^{3}$

$y(4) = \alpha^{-2} + \alpha^{-4}\, 9 + 2\alpha^{1} + 4\alpha^{2} + 8\alpha^{3}.\ h(4) \cdot x(4)$

$= \alpha^{-2} + \alpha^{-3} + 1 + 2\alpha^{1} + 4\alpha^{2} + 8\alpha^{3} + 16\alpha^{4}$

$y(5) = 1 + 2\alpha^{1} + 4\alpha^{2} + 8\alpha^{3} + 16^{4} + 5$

$y(6) = 4\alpha^{2} + 8\alpha^{3} + 16\alpha^{4} + \alpha 5 + \alpha 6$

$y(7) = 8\alpha^{3} + \alpha^{4} + \alpha^{5} + \alpha 6$

$y(8) = 16\alpha^{4} + \alpha^{5} + \alpha 6$

$y(9) = \alpha^{5} + \alpha 6$

$y(10) = \alpha^{6}.$

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Q3 Determine the z-transform of the following signal and sketch it region of convergence (ROC)

(1) $x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n} & n < 0 \end{cases}$

(2) $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n, & n \geq 0 \\ 0 & \text{elsewhere.} \end{cases}$

(1)

Sol

writing in the form of z-transform

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^n z^{-n} - 1$$

Using geomatic series.

$$= \frac{1}{1 - \frac{1}{4} z^{-1}} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^n - 1$$

$$= \frac{1}{1 - \frac{1}{4} z^{-1}} + \frac{1}{1 - \frac{2}{3} z} - 1$$

(16) Abdul Basit 15689

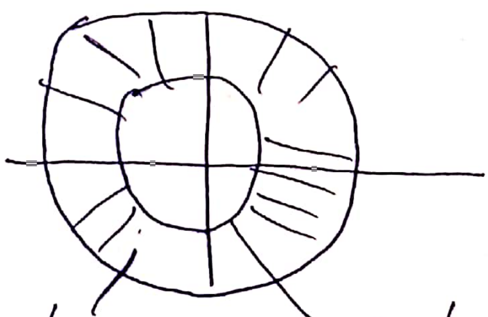
$$= \frac{1-\frac{1}{3}z+1-\frac{1}{4}z^{-1} - 1}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{1-\frac{1}{3}z+1-\frac{1}{4}z^{-1}-(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{\cancel{1}-\cancel{\frac{1}{3}z}+1-\cancel{\frac{1}{4}z^{-1}}-\cancel{1}+\cancel{\frac{1}{3}z}+\cancel{\frac{1}{4}z^{-1}}+\frac{1}{2}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{\frac{13}{12}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

Hence, the ROC is $\frac{1}{4} < |z| < 3$.



$z = \frac{1}{3}$ $z = \frac{1}{4}$

(17) Abdul-Basit 13684.

(h) $x(n)=\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{-n} - \sum_{n=0}^{\infty} 3^n z^{-n}$

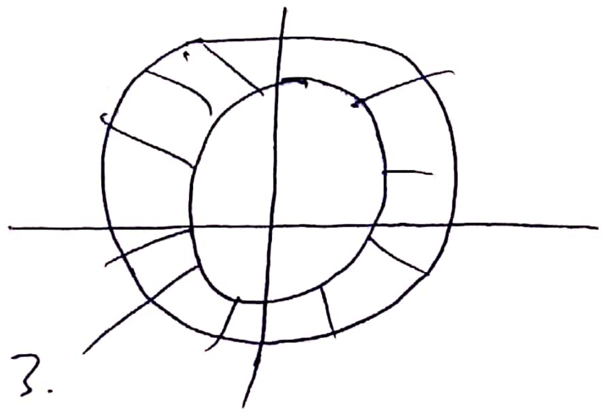
Using geomatric Series to simpfy it

$$= \frac{1}{1-\frac{1}{2} z^{-1}} - \frac{1}{1-3z^{-1}}$$

$$= \frac{\cancel{1}-3z^{-1}-\cancel{1}+\frac{1}{2} z^{-1}}{(1-\frac{1}{2} z^{-1})(1-3z^{-1})}$$

$$= \frac{-\frac{5}{2} z^{-1}}{(1-\frac{1}{2} z^{-1})(1-3z^{-1})}$$

The ROC is $|z|>3$.



Z=3.

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