Department of Electrical Engineering
Mid – Term Assignment Spring 2020
Date: 20/04/2020

Course Details

Course Title: Numerical Analysis **Module:** ____

Instructor: ____ **Total Marks:** 30

Student Details

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Q1.	(a)	1. A 3x3 identity matrix has a total of 3 and ______ Eigen values. a. same b. different c. none d. zero 2. Eigen values of a symmetric matrix are all __________ a. real b. complex c. zero d. positive 3. All of the following are finite difference methods except for __________ a. Jacobi's method b. Newton's backward difference method c. Stirlling formula d. Forward difference method 4. The characteristics polynomial of a 3x3 identity matrix is ________, if x is the Eigen value of the 3x3 identity matrix. a. $(x-1)^3$ b. $(x+1)^3$ c. $x^3 - 1$ d. $x^3 + 1$ 5. Two matrixes with the same characteristic polynomial does not need to be similar a. true b. false 6. Is the determinant of a diagonal matrix the product of the diagonal elements? a. true b. false 7. The Jacobi's method is a method of solving a matrix equation on a matrix that has _____ zeros along its main diagonal. a. No b. At least one c. At least two d. At least three 8. The power method can be used only to find the Eigen value of "A" that is largest in absolute value, we call this Eigen vale the dominant Eigen value of "A". a. true b. false 9. Central difference method is the finite difference method. a. true b. false 10. Iterative algorithms can be more rapid than direct methods. a. true b. false 11. $\Delta f_r = f_{r+1} - f_r$ is known as _________ difference operator. a. forward b. backward c. central d. none	Marks 11 CLO 1
Q2.	(a)	Use bisection method to solve the equation $x^2 - 7 = 0$, Perform four iterations and show all the necessary steps.	Marks 6 CLO 1
Q3.	(a)	Interpolate the value of 0.25 using Newton's forward difference formula. Show all the necessary steps.	Marks 6 CLO 1
	(b)	Use Newton Raphson method to find root of $f(x) = x^3 - 2x + 2$ *with* $x_0 = 0.2$. Perform four iterations.	Marks 6 CLO 1

X	0.2	0.3	0.4	0.5	0.6
F(x)	0.2304	0.2788	0.3222	0.3617	0.3979

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Q1 # multiple choice questions.

(1) A 3X3 identity materix has a total of 3 and ____ Eigen values?

Ans# Same (A).

(2) Eigen values of a symmetric materix are all ____

Ans # $\boxed{A}$ real.

(3) All of the following are finite difference methods except for ____.

Ans # (b) newton backward difference method.

(4) The characteristics polynomial of a 3x3 identity matrix is ____. if x is the Eigen value of the 3x3 identity matrix.

Ans # (C) x^3-1.

(5) two matrix with the same characteristic polynomial does not need to be similar

Ans # (b) false

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⑥ Is the determinant of a diagonal matrix the product of the diagonal element?

Ans #(A) true.

⑦ The Jacobi's method is a method of solving matrix equation on a matrix that has — zeros along its main diagonal.

Ans (A) no

⑧ The power method can be used only to find the Eigen values of 'A' that is largest in absolute values. We call this Eigen value the dominant Eigen value of 'A'.

Ans # (A) true

⑨ Central difference method is the finite difference method

Ans #(A) true

⑩ Iterative algorithms can be more rapid than direct method.

Ans # (B) false

⑪ $\Delta f_r = f_{r+1} - f_r$ is known as — difference operator.

Ans # (A) forward.

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Q2 # Use bisection method to solve the equation $x^2 - 7 = 0$, Perform four iterations and show all the necessary steps?

SOL#

$F(x) = x^2 - 7 = 0$

Step No 1 #

First eq = 0?

~~iteration~~

assume limit [~~upper~~ lower, upper]

$limits = [\underset{l}{1}, \underset{u}{2}]$

$F(1) = (1)^2 - 7 = -6$

$F(2) = (2)^2 - 7 = -3$

$F_1 \times F_2 = -6 \times -3$

$= +18 > 0$

Iteration 1 # New limit $[1, 3]$

$F(3) = (3)^2 - 7 = 2$

$F(1) \times F(3) = -6 \times 2 \quad = -12 < 0$

Step 2 # If $F(l) \times F(u) = Ans < 0$ then Find mid point

↳ if $F(l) \times F(u) = Ans > 0$ then change limit by Finding point

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Find $C = ?$ mid point

$$C = \frac{1+3}{2} = \frac{4}{2} = \boxed{2}$$ pulling in the eq

$$\boxed{F(C) = (2)^2 - 7 = -3}$$

Iteration 2 ## New limit $(2, 3)$

mid point = ?

$$C = \frac{2+3}{2} = \frac{5}{2} = 2.5$$ put in eq

$$F(2.5) = (2.5)^2 - 7 = 6.25 - 7 = \boxed{-0.75}$$

$$F(2.5) \times F(3) = (-0.75) \times 2 = -1.5 < 0$$

Step#3# $[\underset{l}{2.5}, \underset{u}{3}]$

iteration 3 #

$$C = \frac{2.5+3}{2} = \boxed{2.75}$$ put in eq

$$F(C) = (2.75)^2 - 7 = \boxed{-5.625}$$

$$F(C) \times F(U) = -5.625 \times 3 = -4 < 0$$

$[2.75, 3]$

Step 4 #

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iteration 4 #

$$C = \frac{2.75+3}{2} = 2.875$$ put in eq

$F(C) = (2.875)^2 - 7 = 1.265$

$F(C) \times F(3) = 1.265 \times 2 = \boxed{2.5379}$

$F(C) \times F(2.75) = 1.265 \times -5625 = -7,115.6 < 0$

$[2.75, 2.875]$

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Q3(a)# Interpolate the value of 0.25 using newton's forward difference formula. show all the necessary steps.

x	0.2	0.3	0.4	0.5	0.6
F(x)	0.2304	0.2788	0.3222	0.3617	0.3979

Sol#

Step 1# Difference table

x	y	Δ	$\Delta 2$	$\Delta 3$	$\Delta 4$
0.2	0.2304	0.0484	-0.005	+0.0011	0.0005
0.3	0.2788	0.0434	-0.0039	0.0006	
0.4	0.3222	0.0395	-0.0033		
0.5	0.3617	0.0362			
0.6	0.3979				

y_0	Δy_0	$\Delta 2 y_0$	$\Delta 3 y_0$	$\Delta 4 y_0$
0.2304	0.0484	-0.005	+0.0011	0.0005

→ Step No2

[illegible]#

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Step No 3 #

$x = a + nh$

$\Rightarrow x = 0.25$

$a \Rightarrow 0.2$

$h = 0.1 \quad [h = x_2 - x_1]$

putting the values

$x = a + nh$

$0.25 = 0.2 + n(0.1)$

$0.25 - 0.20 = n(0.1)$

Step No 4 $\quad 0.05 = n(0.1)$

Now find $n =$

$$n = \frac{0.05}{0.1} = 0.5$$

$$\boxed{\therefore n = 0.5}$$

~~Step~~ #

$n = 0.5 \quad y_0 = 0.2304, \quad \Delta y_0 = 0.0484 \quad \Delta_2 y_0 = -0.005$

$\Delta_3 y_0 = +0.0011, \quad \Delta_4 y_0 = 0.0005$

using formula

$F(a+nh) =$ ~~f(a) + n Δf(a) + n(n-1)/2!~~

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Step 5 #

putt the values in formula

$$f(x) = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 y_0$$

$$F(X) = 0.2304 + (0.5)(0.0484) + \frac{(0.5)(0.5-1)}{2\times 1}(-0.005) + \frac{0.5(0.5-1)(0.5-2)}{3\times 2\times 1}(+0.0011) + \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{4\times 3\times 2\times 1}(-0.0005)$$

$$F(x) = 0.2304 + 0.0242 + \frac{(-0.25)}{2}\,\frac{(-0.005)}{1} + \frac{(0.375)}{6}\,\frac{(0.0011)}{1} + \frac{(-0.9375)}{24}\,\frac{(-0.0005)}{1}$$

$$= 0.2304 + 0.0242 + 0.000625 + 0.00006875 + 0.0000195313$$

$$= \boxed{0.2559320313} \text{ Ans}$$

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Q3(b)# Use newton Raphson method to find root of $f(x) = x^3 - 2x + 2$ with $x_0 = 0.2$ Perform four iterations?

Sol# Using newton Raphson formula

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

put $f(x_i)$ and $f'(x_i)$ values

$$x_{i+1} = \frac{x_i}{1} - \frac{(x_i^3 - 2x_i + 2)}{(3x_i^2 - 2)}$$

$$= \frac{x_i(3x_i^2 - 2) - (x_i^3 - 2x_i + 2)}{3x_i^2 - 2}$$

$$= \frac{3x_i^3 - \cancel{2x_i} - x_i^3 + \cancel{2x_i} - 2}{3x_i^2 - 2}$$

$$x_{i+1} = \frac{2x_i^3 - 2}{3x_i^2 - 2}$$

iteration 1 #

$x_0 = 0.2$

$$= \frac{2(0.2)^3 - 2}{3(0.2)^2 - 2}$$

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$= \frac{\cancel{-} 1.984}{\cancel{-} 1.88} = \boxed{1.0553}$

iteration 2 #

$x_0 = 1.0553$

$= \frac{2(x_i^3) - 2}{3(x_i^2) - 2}$

$= \frac{2(1.0553)^3 - 2}{3(1.0553)^2 - 2} = \frac{0.3504}{1.3409} = \boxed{0.2613}$

iteration 3 #

$x_0 = 0.2613$

$= \frac{2x_i^3 - 2}{3x_i^2 - 2} = \frac{2(0.2613)^3 - 2}{3(0.2613)^2 - 2}$

$= \frac{\cancel{-} 1.9643}{\cancel{-} 1.7951} = \boxed{1.0942}$

iteration 4 #

$x_0 = 1.0942$

$= \frac{2x_i^3 - 2}{3x_i^2 - 2} = \frac{2(1.0942)^3 - 2}{3(1.0942)^2 - 2}$

$= \frac{0.6201}{1.5918} = \boxed{0.3895}$ Ans.