FINAL TERM

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SECTION :- B

SUBJECT :- STRUCTURAL ANALYSIS

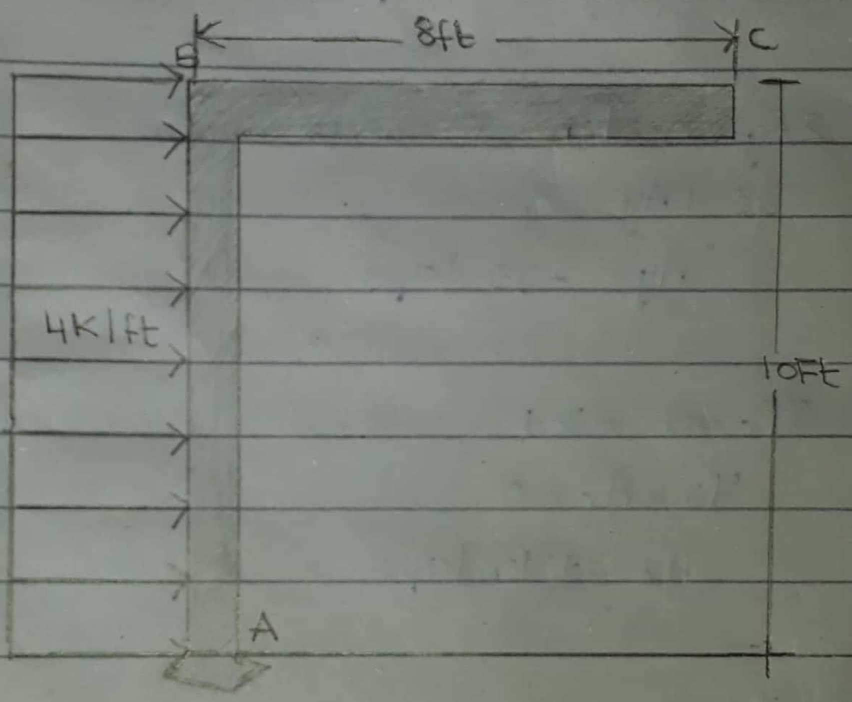
SUBMITTED TO :- AMJAD ISLAM

DATE :- 26-JUNE 2020

DEPARTMENT :- CIVIL ENGINEERING

QUESTION :- 01

Determine the vertical Displacement of free end point C on the .... virtual work.



Given that :-

$E = 29(10^3)$ ksi

$I = 600 \text{ in}^4$

Required :-

Virtual work, $\Delta_C = ?$

For Reaction :-

$\Sigma M_A = 0$

$-4(10)(5) + C_y(8) = 0$

$C_y = 25$ kips

$\Sigma f_y = 0 \uparrow +$

$25 + A_y = 0$

$A_y = -25$ kips

$\Sigma f_x = 0 \rightarrow +$

$40 - A_x = 0$

$A_x = +40$ kips

Real Moments :-

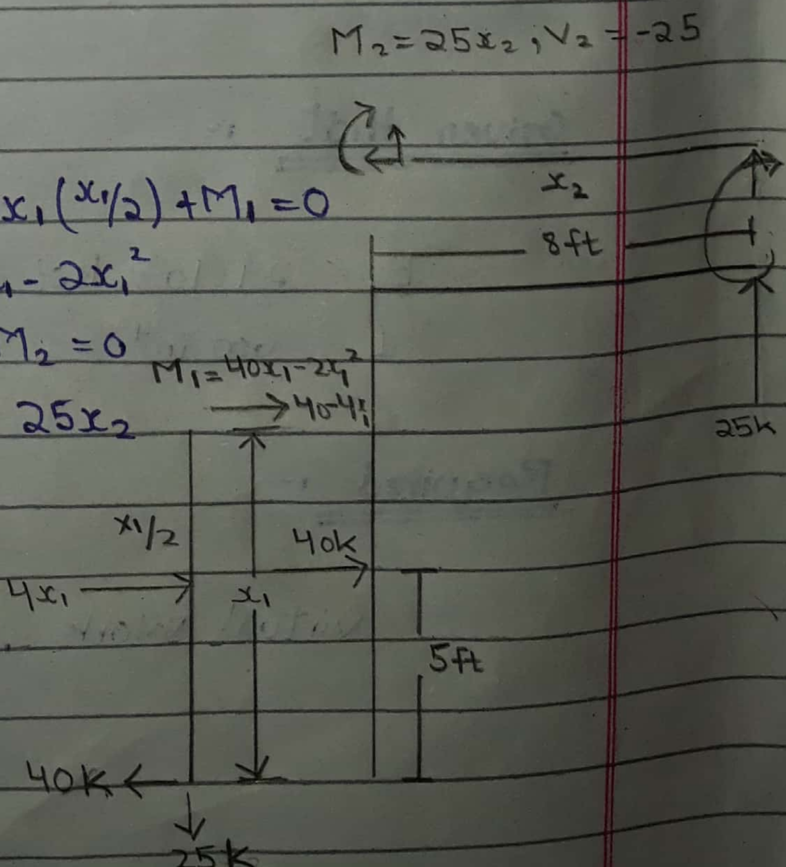
$\Sigma M_1 = 0$

$-40(x_1) + 4x_1(x_1/2) + M_1 = 0$

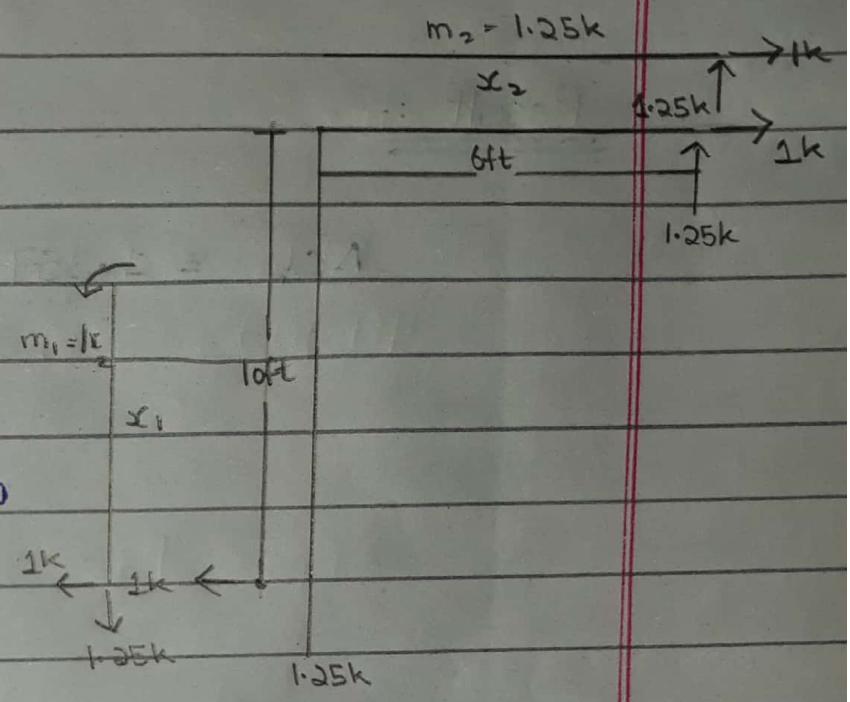
$M_1 = 40x_1 - 2x_1^2$

$-25x_2 + M_2 = 0$

$M_2 = 25x_2$



Virtual Moments :-



$\Sigma m_1 = 0$

$-1(x_1) + m_1 = 0$

$m_1 = 1x_1$

$-m_2 + 1.25\, x_2 = 0$

$m_2 = 1.25 x_2$

Now From virtual work equation :-

$$1k \cdot \Delta C_h = \int_0^l m\, M dx / EI$$

$$1k \cdot \Delta C = \int_0^{18} \frac{(40x_1 - 2x_1^2)(1x_1)\, dx}{EI}$$

$$\Delta C_h = \frac{8333.3}{EI} + \frac{5333.3}{EI}$$

$$\Delta C_h = \frac{13666.7}{EI}\ k^2 \cdot ft^3$$

$$\Delta C_h = \frac{13666.7\ k^2 . ft\ (12^3 in^3 / 1 ft^3)}{[29 \times 10^3\ k/in^2](600)}$$

$$\Delta C_h = 1.357 \text{ inch}$$

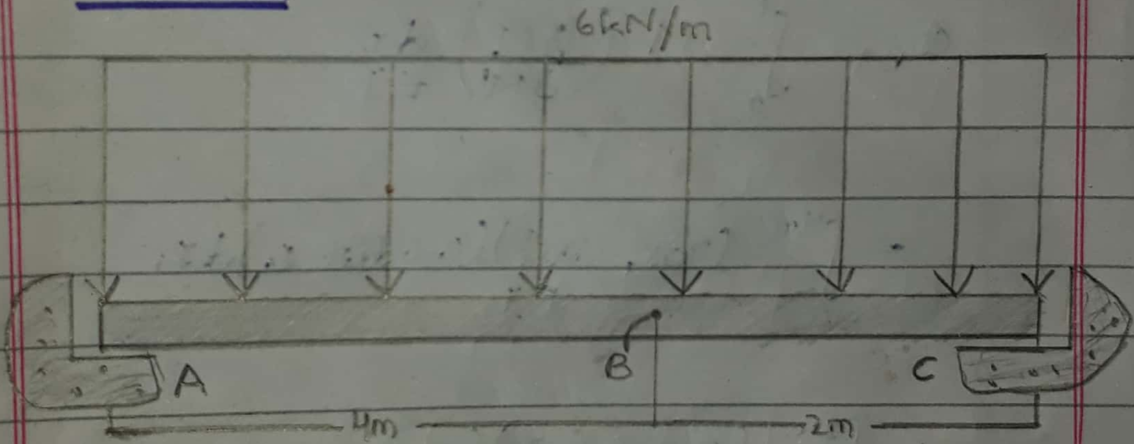
Result :-

$\Delta C_n = 1.357$ inch

QUESTION:- 02

Determine the slope and displacement at point B. Assume the support at A is a pin and C is a roller. Take E = 200 Gpa $I = 60(10)^6 mm^4$. Use Castigliano's Theorem.

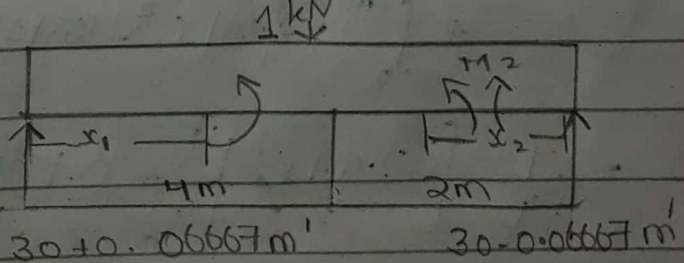
Given :-



Required:-

we have to find the slope and displacement.

Solution:-



$M_1 = (30 + 0.06667 M')\, x_1 - 2x_1^2$

$M_2 = (30 - 0.06667 M')\, x_2 - 2x_2^2$

Now,

$\frac{\partial M_1}{\partial M'} = 0.06667\, x_1$ and $\frac{\partial M_2}{\partial M'} = 0.06667\, x_2$

Set $M' = 0$

Now $M_1 = (30x_1 - 2x_1^2)$ k.ft

& $M_2 = (30x_2 - 2x_2^2)$ k.ft

Thus,

$$\theta_B = \int_0^L M \left(\frac{\partial M}{\partial M'}\right) \frac{dx}{EI}$$

$$= \int_0^{4m} \frac{(30x_1 - 2x_1^2)(0.06667\, x_1)\, dx_1}{EI}$$

$$+ \int_0^{2m} \frac{(30x_2 - 2x_2^2)(0.06667\, x_2)\, dx_2}{EI}$$

$$= \frac{1}{EI}\left[\int_0^{4m} (2.001\, x_1^2 - 0.1334\, x_1^3)\, dx_1 \right.$$

$$\left. + \int_0^{2m} (2.001\, x_2^2 - 0.1334\, x_2^2)\, dx_2\right]$$

$$= \frac{1}{EI}\left[\frac{2.001\, x_1^2}{3}\Bigg|_0^4 - \frac{0.1334\, x_1^3}{4}\Bigg|_0^2\right.$$

$$\left. + \frac{2.001\, x_2^2}{3}\Bigg|_0^2 - \frac{0.1334\, x_2^3}{4}\Bigg|_0^2\right]$$

$$= \frac{1}{EI}\left[0.667\, x_1^3 \Big|_0^4 - 0.03335\, x_1^4 \Big|_0^4 + 0.667\, x_2^3 \Big|_0^2 - 0.03335\, x_2^4 \Big|_0^2\right]$$

$$\theta_B = \frac{1}{EI}\left[(42.688 - 8.5376) + (5.336 - 0.5336)\right]$$

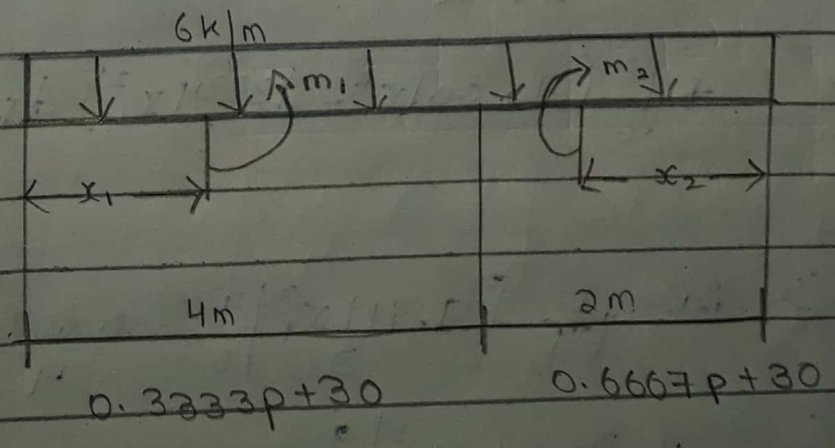
$$\Rightarrow \theta_B = \frac{38.9504}{EI}$$

$$\Rightarrow \theta_B = \frac{38.9504}{(200 \times 10^9) \times (60 \times 10^6 \text{ mm}^4)}$$

$$\Rightarrow \theta_B = \frac{38.9504}{(200 \times 10^9)(6 \times 10^{-5})}$$

$$\theta_B = 0.03245 \times 10^{-4} \text{ radian}$$

Now displacement



Now $M_1 = (0.33p + 30)\,x_1 - 2x_1^2$

$M_2 = (0.6667p + 30)\,x_2 - 2x_2^2$

Now $\dfrac{\partial M_1}{\partial p} = 0.3333\,x_1$

And, $\dfrac{\partial M_2}{\partial p} = 0.6667\,x_2$

Set $p = 0$

$M_1 = (30x_1 - 2x_1^2)$ and $M_2 = (30x_2 - 2x_2^2)$

$$\Delta B = \int_0^l M\left(\frac{\partial M}{\partial p}\right)\frac{dx}{EI}$$

$$\Delta B = \frac{1}{EI}\left[\int_0^4 (30x_1 - 2x_1^2)\,0.3333\,x_1\,dx_1 + \int_0^2 (30x_2 - 2x_2^2)\,0.6667\,dx_2\right]$$

$$\Rightarrow \Delta B = \frac{1}{EI}\left[\int_0^4 (9.999\,x_1^2 - 0.6666\,x_1^3)\,dx_1 + \int_0^2 (20.001x_1^2 - 1.3334\,x_1^3)\,dx_2\right]$$

$$\Rightarrow \Delta B = \frac{1}{EI}\left[\frac{9.999\,x_1^3}{3}\Big|_0^4 - \frac{0.6666\,x_1^4}{4}\Big|_0^4 + \frac{20.001\,x_2^3}{3}\Big|_0^2 - \frac{1.3334\,x_2^4}{4}\Big|_0^2\right]$$

$$\Delta B = \frac{1}{EI}\left[(213.33 - 42.6624) + (53.336 - 5.336)\right]$$

$$\Delta B = \frac{218.6676}{(200 \times 10^{9}) \times (6 \times 10^{-5})}$$

$$\Delta B - 0.1822 \times 10^{-4} \text{ ft}$$

$$\Delta B = 2.186 \times 10^{-4} \text{ inch}$$

Result :-

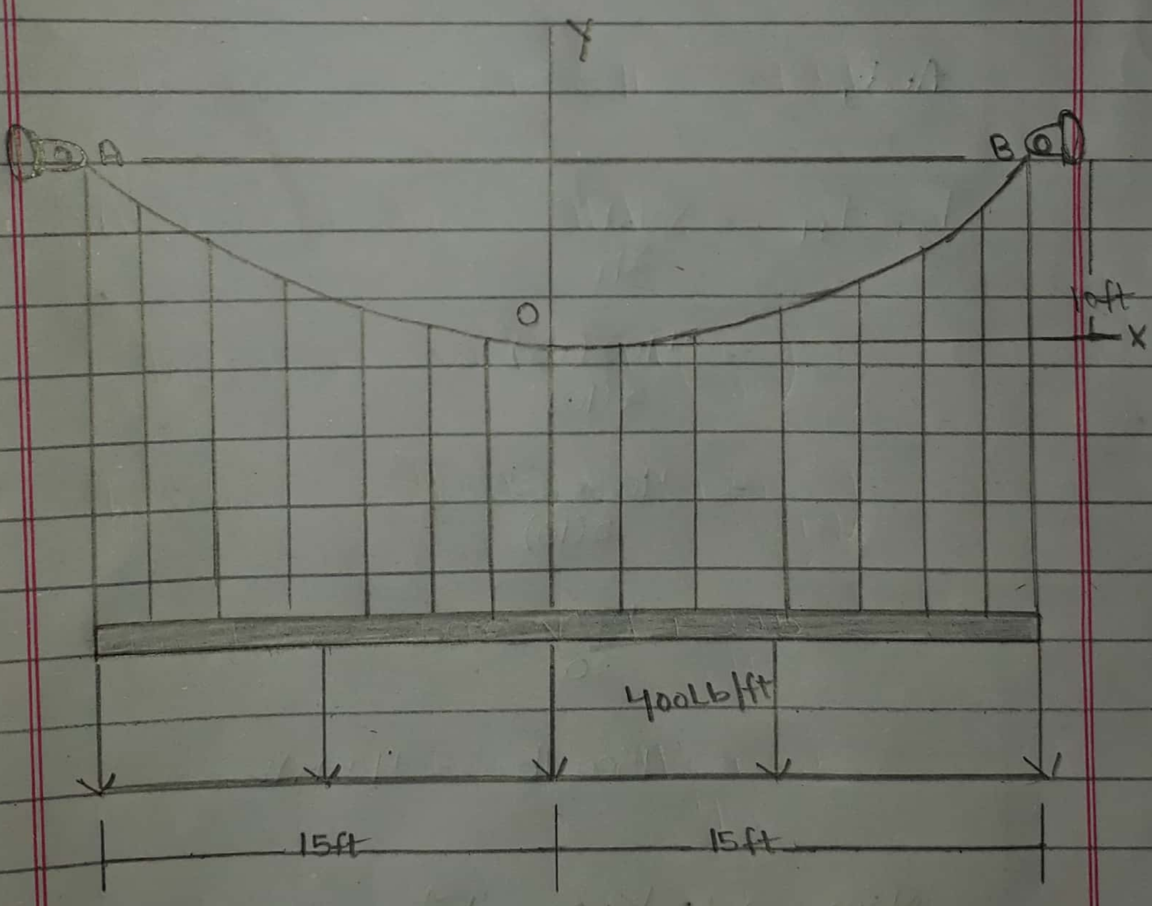
Hence

$$\theta_B = 0.03245 \times 10^{-4}$$

$$\Delta B = 2.186 \times 10^{-4}$$

QUESTION :- 03

The cable is subjected to the uniform Loading. If the slope of the cable at point O is zero, Determine the equation of the curve and the force in the cable at O and B.



Given that :-

slope at point O is Zero.

Required :- Equation of Curve & force in the Cable at O & B.

Solution :-

As we know that,

$$y = \frac{h}{L^2} x^2 = \frac{10}{(15)^2} x^2$$

$$y = 0.0444\, x^2$$

And,

$$T_0 = F_H = \frac{W_0 L^2}{2h}$$

$$= \frac{400 (15)^2}{2(10)}$$

$$= \frac{400 \times 225}{2(10)}$$

$$= \frac{90000}{20}$$

$$= 4500 \text{ lb} = 4.5 \text{ Klb}$$

Now, we know that

$$T_H = T_{max} = \sqrt{(F_H)^2 + (W_0 L)^2}$$

$$T_H = T_{max} = \sqrt{(4500)^2 + [(400)(15)]^2}$$

$$T_H = T_{max} = \sqrt{(4500)^2 + (6000)^2}$$

$$T_H = T_{max} = \sqrt{20250000 + 36000000}$$

$$T_H = T_{max} = \sqrt{56250000}$$

$$T_H = T_{max} = 7500 \text{ lb} = 7.5 \text{ klb}$$

Also, We know that

$$T_H = T_{max} = W_o L \sqrt{1 + \left(\frac{1}{2h}\right)^2}$$

$$T_H = T_{max} = 400(15)\sqrt{1 + \left(\frac{15}{2(10)}\right)^2}$$

$$T_H = T_{max} = 6000\sqrt{1 + \frac{225}{400}}$$

$$T_H = T_{max} = 6000\sqrt{\frac{400+225}{400}}$$

$$T_H = T_{max} = 6000\sqrt{\frac{625}{400}}$$

$$T_H = T_{max} = 6000\sqrt{1.5625}$$

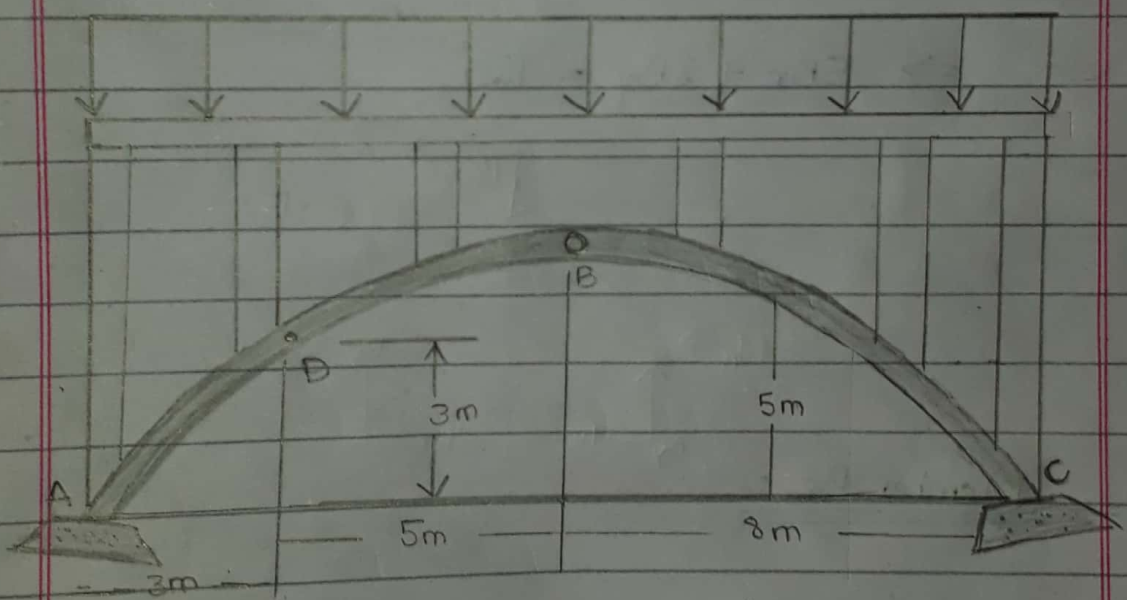
$$T_H = T_{max} = 6000(1.25)$$

$$T_H = T_{max} = 7500 \text{ lb}$$

OR $T_H = T_{max} = 7500 lb = 7.5 klb$

QUESTION :- 04

THe three - hinged spandrel arch is Subjected to the Uniform Load of 30 kN/m. Determine the internal moment in the arch at point D.



Given that :-

Uniformly Distributed Load of 30 kN/m.

Required :-

Internal Moment in the arch at point D.

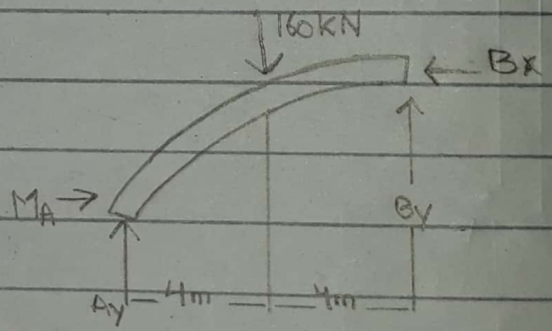
Solution :-

Member AB :-

$\circlearrowright + \Sigma M_A = 0$

$\Rightarrow B_x(5) + B_y(8) - 240(4) = 0$

$\Rightarrow 5B_x + 8B_y = 960$



Member BC :-

$\circlearrowright + M_C = 0$

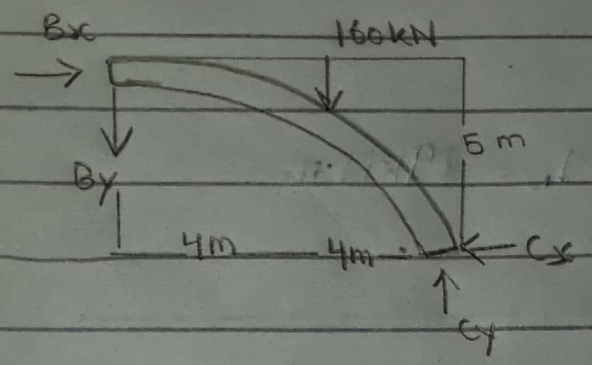
$\Rightarrow -B_x(5) + B_y(8) + (240)4 = 0$

As $B_y = 0$

$\Rightarrow -5B_x + 0 + 960 = 0$

$\Rightarrow 5B_x = 960$

$\Rightarrow B_x = 192$



Now Segment BD :-

$\circlearrowright M_D = 0$

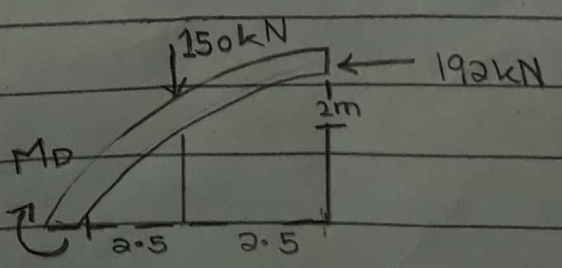
$\Rightarrow (B_x)(2) - (150 \times 2.5) - M_D = 0$

$(192 \times 2) - (375) - M_D = 0$

$384 - 375 - M_D = 0$

$9 - M_D = 0$

$M_D = 9 \text{ kN·m}$



Result :-

Hence $M_O = 9\,kN\cdot m$