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Section : "B"

Paper : Differential equation:

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Q No. 1

$$dy/dt = e^{y-t}\sec(y)(1+t^2) \quad y(0)=0$$

Sol:-

$$dy/dt = e^{y-t}\sec(y)(1+t^2). \quad y(0)=0$$

so $y=0, x=0$

$$dy = e^{y}e^{-t}\sec(y)(1+t^2)dt$$

$$\frac{1}{e^{y}.\sec(y)} = (1+t^2)e^{-t}dt$$

As $\cos(y) = \frac{1}{\sec(y)}$

$$\int e^{-y}\cos y\,dy = \int(1+t^2)\,dt$$

using integration by parts

$$e^{-y}\int\cos y\,dx - \int\left(\int\cos y \cdot \frac{d}{dy}e^{-y}\right) = (1+t^2)$$

$$\int e^{-t} - \int\left(\int e^{-t}\frac{d}{dt}(1+t^2)\right) \rightarrow eq(i)$$

L.H.S

$$e^{-y}\int\cos y\,dx - \int\left(\int\sec y \cdot \frac{d}{dy}e^{-y}\right)$$

$$e^{-y}\sin y - \int(\sin y . e^{-y}(-1)$$

$$e^{-y}\sin y + \int(\sin y . e^{-y})\; e^{-y}\sin y + \int(e^{-y}\sin y)$$

Again using integration by parts.

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$e^{-y}\sin y + e^{-y}(-\cos y) - \int(\int \sin y \frac{d}{dy} e^{-y})$

$e^{-y}\sin y + e^{-y}(-\cos y) - \int(-\cos \frac{e^{-y}}{-1})$

$e^{-y}\sin y - e^{-y}\cos y \int(\cos y\, e^{-y})$

Since ~~$\int(\cos e^{-y})$~~ $\int(\cos e^{-y}) = L\cdot H\cdot S$

Since its again same to the first one so L.H.S will become

$L\cdot H\cdot S = e^{-y}(\sin y - \cos y) - L\cdot H\cdot S$

$2L\cdot H\cdot S = e^{-y}(\sin y - \cos y)$

$L\cdot H\cdot S = \frac{e^{-y}(\sin y - \cos y)}{2}$

Now taking R.H.S

$\int(1+t^2)e^{-t}dt$

$= (1+t^2)\int e^{-t} - \int(\int e^{-t} \frac{d}{dt}(1+t^2))$

$= (1+t^2)e^{-t} - \int(-e^{-t}(2t))$

$-(1+t^2)e^{-t} + \int(2t)e^{-t})$

Again using integration by Parts

$-(1+t^2)e^{-t} + (2t\int e^{-t} - \int(\int e^{-t} \frac{d}{dt}2t))$

$-(1+t^2)e^{-t} + (-2te^{-t} - \int(-e^{-t}2)$

$-(1+t^2)e^{-t} + (-2t\, e^{-t} + \int (2e^{-t})$

$-(1+t^2)e^{-t} + (-2t\,e^{-t} - 2e^{-t}) + c.$

$\Rightarrow -(1+t^2)e^{-t} - 2t\,e^{-t} - 2e^{-t} + c$

$\Rightarrow -e^{-t} - e^{-t}t^2 - 2te^{-t} - 2e^{-t} + c$

$\Rightarrow -(t^2+2t+3)e^{-t} + c = R.H.S$

Now take L.H.S = R.H.S

$$\frac{e^{-y}(\sin y - \cos y)}{2} = -(t^2+2t+3)e^{-t} + c$$

we know that

$t = 0 \qquad y = 0$

Put it above

$\Rightarrow \frac{1}{t}(0-1) = -3 + c$

$c = 5/2$

Put value of c

$$\frac{e^{-y}}{2}(\sin y - \cos y) = -(x^2+2t+3)e^{-t} + 5/2.$$

Q No. 2

Sol:- $(\sqrt{x+y} + \sqrt{x-y})dx - (\sqrt{x+y} - \sqrt{x-y}) = 0$

$$\frac{dy}{dx} = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} \longrightarrow (1)$$

This is homogenous Differential eq x and y To solve this put

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Thus eq is becomes

$$v + x \cdot \frac{dv}{dx} = \frac{\sqrt{x+vx} + \sqrt{x-vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{1+\cancel{v}+1-\cancel{v}+\cancel{2}\sqrt{1-v^2}}{\cancel{2}v}$$

$$v + x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}-v^2}{v}$$

$$x\frac{dv}{dx} = \sqrt{1-v^2}\,\frac{(1+\sqrt{1-v^2}}{x}$$

$$\frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \frac{dx}{x}$$

Taking Integral on b/s

$$\int \frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \int \frac{dx}{x}$$

Put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \frac{1}{2}(1-v^2)^{-1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int -\frac{dt}{t} = \int \frac{dx}{x}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln(1+\sqrt{1-v^2}) = -\ln cx$$

$$\ln(1+\sqrt{1-v^2}) = -\ln cx$$

$$\cancel{\ln}(1+\sqrt{1-v^2}) = \cancel{\ln}(cx)^{-1}$$

$$1+\sqrt{1-v^2} = \frac{1}{cx}$$

$$1+\sqrt{1-\frac{y^2}{x^2}} = \frac{1}{cx}$$

$$1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{cx}$$

$$x+\sqrt{x^2-y^2} = \frac{1}{c}$$

$$x+\sqrt{x^2-y^2} = c_1 \quad \therefore \frac{1}{c} = c_1$$

which is required Solution.

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QNo. 3

$$(D^4+D^2)y = 3x^2+4\sin x - 8\cos x$$

Solution:-

$$(D^4+D^2)y = 3x^2+4\sin x - 8\cos x$$

$$\Rightarrow f(D)y = f(x)$$

As it is non-homogenous linear equation so solution will be

$y = y_c + y_P \rightarrow (i)$

Complementary solution y_c

$D^4 - D^2 = 0 \Rightarrow D^2(D^2+1) = 0$

Either $D^2 = 0 \Rightarrow \boxed{D=0}$

$D^2 + 1 = 0 \Rightarrow D^2 = -1$

$D = \sqrt{-1} = \boxed{D = i}$ or $D = \boxed{0+i}$

Roots are real & complex

$y_c = C_1 e^{0x} + e^{0x}(C_2 \cos x + C_3 \sin x)$

$y_c = C_1 + C_2 \cos x + C_3 \sin x$

$y_P = \frac{1}{f(D)} F(x)$

$y_P = \frac{1}{D^4 + D^2}(3x^2 + 4\sin x - 2\cos x)$

$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$

$f(D) = D^4 + D^2$

at $D = 0 \Rightarrow f(D) = 0$

So $f(D) = 4D^3 - 2D$

Now also for $D = 0 \Rightarrow f'(D) = 0$

again differentating

$f''(D)$, $12D + 2$

So for $D = 0$

$f''(0) . 12(0) + 2 = 2$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow y_P = \frac{x^2\, 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2} \cdot 2\cos x$$

Putting $D = 0$ in whole

$$y_P = \frac{x^2\, 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} - \frac{2x\cos x}{12(0)+2}$$

$$y_P = \frac{3x^4}{2} + \frac{4x^2\sin x}{2} - \frac{2x^2\cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

So putting in eq (i)

$$y = c_1 + c_2\cos x + c_3\cos x + \tfrac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

$$\boxed{y = c_1 + (c_2 - x^2)\cos x + (c_3 + 2x^2)\sin x + \tfrac{3}{2}x^4}$$

THE END