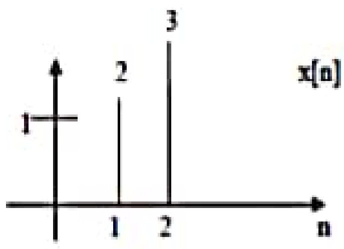
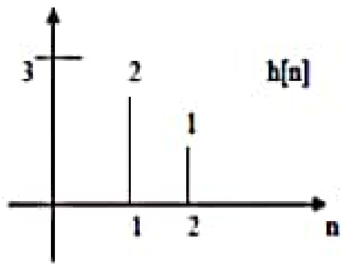
Department of Electrical Engineering
Final term exam
Date: 23/09/2020

Course Details

Course Title: Signals & Systems **Module:** 04
Instructor: ______ **Total Marks:** 50

Student Details

Name: ______ **Student ID:** ______

Q1.		Identify the basic difference between a periodic and an aperiodic signal using examples.	Marks 06 CLO 1
Q2.		$f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ \pi & 0 \le x \le \pi \end{cases}$ Retrieve the Fourier series for the given function.	Marks 12 CLO 3
Q3.		If $X(z) = 2z^2 + 2z / (z^2 + 2z - 3)$ Retrieve x[n] using inverse Z-transform method.	Marks 10 CLO 3
Q4.		If $x[n] = 4\delta[n] - 3\delta[n-1] + 4\delta[n-2]$ $h[n] = 2\delta[n-1] - 3\delta[n-2] + 2\delta[n-3]$ Produce Y(z) and y[n]	Marks 10 CLO 3
Q5.		Evaluate y[n] using convolution summation.  	Marks 12 CLO 2

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Q1

Periodic signal

Ans

signal that repeat it pattern over a period is called periodic signal.

example

periodic signals include the sinusoidal signals and periodically repeated non-sinusoidal signal such as the rectangular pulse sequences used in radar.

P#1

aperiodic signal

Ans

A signal that does not repeats its pattern over a period is called aperiodic signal.

example

non-periodic signals include speech waveforms and random signals arising from unpredictable disturbances of all kinds.

Q_2 $f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ \pi & 0 \le x \le \pi \end{cases}$

Ans

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx \right]$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} \pi\,dx \right]$$

$$= \frac{1}{2\pi} \left[0 + \pi x \Big|_{0}^{\pi} \right]$$

$$= \frac{1}{2\pi} \left[\pi(\pi - 0) \right]$$

$$a_0 = \frac{1}{2\cancel{\pi}} \times \pi^{\cancel{2}}$$

$$a_0 = \frac{\pi}{2}.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0}(0)\cos nx\,dx + \int_{0}^{\pi}\pi\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[0 + \int_{0}^{\pi}\pi\cos nx\,ds\right]$$

$$= \frac{1}{\pi}\left[\pi\int_{0}^{\pi}\cos nx\,dx\right]$$

$$= \frac{1}{\cancel{\pi}}\left[\cancel{\pi}\,\frac{\sin nx}{n}\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{n}\left[\sin n(\pi) - \sin n(0)\right]$$

$$= \frac{1}{n}\left[0 - 0\right]$$

$$a_n = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx\,dx \int_{0}^{\pi} f(x)\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} (0)\sin nx\,dx + \int_{0}^{\pi} \pi\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[0 + \pi\int_{0}^{\pi} \sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\pi\left(\frac{-\cos nx}{n}\right)\Bigg|_{0}^{\pi}\right]$$

$$= \frac{-\pi}{n\pi}\left[\cos nx\Big|_{0}^{\pi}\right]$$

$$= -\frac{1}{n}\left[\cos n(\pi) - \cos n(0)\right]$$

$$b_n = \begin{cases} 0 & \text{if } n \text{ even} \\ \frac{2}{n} & \text{if } n \text{ odd} \end{cases}$$

$$f(x) = a_0 + a_1\cos x + a_2\cos 2x + a_3\cos 3x + \ldots\ldots$$

$$b_1\sin x + b_2\sin 2x + b_3\sin 3x + \ldots\ldots$$

$$f(x) = \frac{\pi}{2} + 2\sin x + \frac{2}{3}\sin 3x.$$

Ans

Given that :

Q3

Ans $x(z) = \frac{2z^2+2z}{(z^2+2z-3)}$

$x(z) = \frac{2z(z+1)}{z^2+3z-z-3}$ Taking "2z" common

$X(z) = \frac{2z(z+1)}{z(z+3)-1(z+3)}$ "simplify"

$\frac{X(z)}{z} \quad \frac{2(z+1)}{(z+3)(z-1)}$

or we can write.

$\frac{2(z+1)}{z^2+2z-3} = \frac{A}{(z+3)} + \frac{B}{(z-1)}$

Putting.

$2(z+1) = A(z-1)+B(z+3) \Rightarrow$ eq (1)

Now putting $z=1$ in eq (1)

P#6

$2(1+1) = B(1+3)$

$4 = 4B$

$\frac{\cancel{4}}{\cancel{4}} = \frac{\cancel{4}B}{\cancel{4}}$

$1 = B$

$B = 1$

Now putting $z = -3$ in eq (1)

$2(-3+1) = A(-3-1)$

$-4 = -4A$

$A = 1$

Now we will put A and B in eq (1)

$$\frac{2(z+1)}{(z+3)(z-1)} = \frac{1}{z+3} + \frac{1}{z-1}$$

$$X(z) = \frac{z}{z+3} + \frac{z}{(z-1)}$$

$$X(z) = \frac{z}{z+3} + \frac{z}{z-1}$$

inverse z-Transform

$$X(z) = u[3] + 1(-1)^K$$

P # 7

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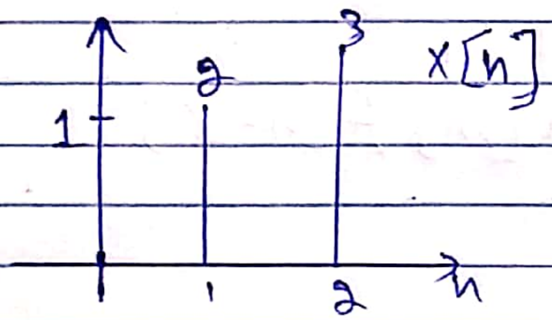
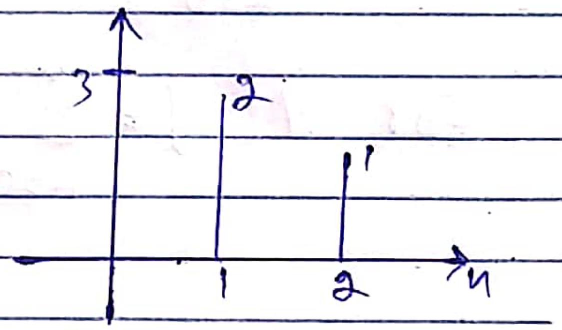
Q5 Evaluate $y[n]$ using Convolution summation.





Ans The summation is called the convolution sum of sequence $x[n]$ and $h[n]$ and represented compactly

$$y[n] = x[n] * h[n]$$

As we know

$$x[n] = x[n] + 2[x-1] + 3x[1-2]$$

and

$$y[n] = 3x[n] + 2x[n-1] + x[n-2]$$

$$x[n] = x[0]\,\delta[n] + x[1]\,\delta[n-1] + x[2]\,\delta[n-2]$$

$y[n] = x[0]\, f[n] + x[1]\, f[n-1] + x[2]\, f$
$(n[-2]$

$$x[n] = \sum_{K=0}^{2} x[m]\, f[n-K]$$

$$\text{for } y[n] = \sum_{K=0}^{2} x[m]\, f[n-K]$$

Ans

Q4 $x[n] = 4\delta[n] - 3\delta[n-1] + 4\delta[n-2]$

$h[n] = 2\delta[n-1] - 3\delta[n-2] + 2[n-3]$

Ans

$x[n] = 4\delta[n] - 3\delta[n-1] + 4\delta[n-2]$

$h[n] = 2\delta[n-1] - 3\delta[n-2] + 2\delta[n-3]$

$y(z)$ $y[n] = ?$

$X(z) = 4 - 3z^{-1} + 4z^{-2}$

$h[z] = 2z^{-1} - 3z^{-2} + 2z^{-3}$

Now $H(z) : X(z)$

$y(z) = H(z) : X(z)$

$= (2z^{-1}) - 2z^{-2} + 2z^{-3})(4 - 3z^{-1} + 4z^{-2})$

$y(z) = 8z^{-1} - 6z^{-2} + 8z^{-3} - 12z^{-2} + 9z^{-3}$
$- 12z^{4} + 8z^{-3} - 6z^{-4} + 8z^{-3}$

$$y(z) = 8z^{-1} - 6z^{-2} - 12z^{-2} + 8z^{-3} + 9z^{-3} + 8z^{-3} - 12z^{-4} - 6z^{-4} + 8z^{-5}$$

$$= 8z^{-1} - 18z^{-2} + 25z^{-3} - 18z^{-4} + 8z^{-3}$$

So

$$y[n] = 8\delta[n-1] - 18\delta[n-2] + 25\delta[n-3] - 18\delta[n-4] + 8\delta[n-5]$$