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PaPER DE

Differential equation in mathematics.

a differential equation is an equation that relates one or more functions and their derivatives

e.g # 1

Find the solution of $y = 5$ where $x = 0, y = 2$

Sol $y = 5$

$dy = 5dx$

$y = 5x + K$

Applying the secondary condition $x = 0, y = 2$

$K = 2$

$y = 5x + 2$

e.g # 2

$y''' = 0$

when $y(0) = 3$ $y'(1) = 4$ $y''(2) = 6$

since $y''' = 0$

$y'' = a$

when a is constant integrating again give

Now, $y(0)=3$ gives $c=3$

$y'(2)=6$ gives $A=6$

$y'(1)=4$ ~~gives~~

$y=3x^2-2x+3$

~~The solution by differentiation~~

$y'=6x-2$

$y'(1)=6(1)-2=4$

$y''=6$

$y'''=0$

b. Define a separable differential equation

Ans Separable D.E

1) solve the following IVP using separable D.E and find the interval of validity the solution

Q1 $y' = xy^3/\sqrt{1+x^2}$ $y(0) = -1$

solution

$y^{-3}\,dy = x\,(1+x^2)^{-1/2}\,dx$

$\int y^{-3}\,dy = \int x\,(1+x^2)^{-1/2}\,dx$

$-1/2y^2 = \sqrt{1+x^2} + C$

$-1/2 = \sqrt{1} + C$

$C = 3/2$

$1/-2y^2 = \sqrt{1+x} - 3/2$

$1/y^2 = 3-2\sqrt{1+x^2}$

~~$y^2 = 1/3-2\sqrt{}$~~

$y^2 = \dfrac{1}{3-2\sqrt{1+x^2}}$

$y(x) = -\dfrac{1}{\sqrt{3-2\sqrt{1+x^2}}}$

Now finding interval of validity

$3-2\sqrt{1+x^2} > 0$

$3 > 2\sqrt{1+x^2}$

$9 > 4(1+x^2)$

$9/4 > 1+x^2$

$5/4 > x^2$

$\sqrt{5/2} < x < \sqrt{5/2}$

$x=0$

Interval of validity

Ans

b $y' = e^{-y}(2x-4)$ $y(5)=0$

Multiply by e^{y} eq by dx

$e^{y}\,dy = (2x-4)\,dx$

$\int e^{y}\,dy = \int (2x-4)\,dx$

$e^{y} = x^2 + 4x + c$

$y = \ln(x^2 + 4x + c)$

$y(5) = \ln((5)^2 - 4(5) + c)$

$\ln(5+c) = 0$

$5+c = 1$

$c = -4$

$\therefore y = \ln(x^2 - 4x - 4)$ Ans

Q2 Solve The following IVP using linear Differential method

i) explain The steps for solving linear Differential equation

Steps

~~Here~~

* substitute $y = uv$ and
$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

into
$$\frac{dy}{dx} + P(x)y = Q(x)$$

* Factor The Parts involving

* Put The v Team equal to zero (This gives a differential equation in v and x which can be solved in The next step)

* solve using separation of variables of Find u

* 5. Substitute u back into The equation we got at steps

* solve That to find v

* Finally, substitute u and v into y = uv to get our solution

11 $\cos(x)\,y' + \sin(x)y = 2\cos^2(x) - 1 \quad y\left(\frac{\pi}{4}\right) = 3\sqrt{2}$

$y' + \frac{\sin(x)}{\cos(x)} y = 2\cos^2(x)\sin(x) - \frac{1}{\cos x}$

$y' + \tan(x) y = 2\cos^2(x)\sin x - \sec(x)$

$\mu(t) = e^{\int \tan(x)\,dx} = e^{\ln \sec(x)} = e^{\sin x} = \sec$

$\int \tan(x)\,dx = -\ln|\cos(x)| = \ln \cos(x)|^{-1} = \ln \sec$

$\sec(x)y' + \sec(x)\tan(x)y = 2\sec(x)\cos^2(x)$
$\sin(x) - \sec^2(x)$

$(\sec(x)y)' = 2\cos(x)\sin(x) - \sec^2(x)$

integ b/s

$\int (\sec(x)y(x))'\,dx = \int 2\cos(x)\sin(x) - \sec^2(x)\,dx$

$\sec(x)y(x) = \int \sin(2x) - \sec^2(x)\,dx$

$\sec(x)y(x) = -\frac{1}{2}\cos(2x) - \tan(x) + c$

$y(x) = -\frac{1}{2}\cos(x)\cos(2x) - \cos(x)\tan(x)$

$+ \cos(x) = -\frac{1}{2}\cos(x)\cos(2x) - \sin(x) + c\cos(x)$

$3\sqrt{2} = \frac{\sqrt{2}}{2} + c\frac{\sqrt{2}}{2}$

$c = 7$

$y(x) = \frac{1}{2}\cos(x)\cos(2x) - \sin(x) + 7\cos(x)$

Q3 Solve the following IVP the exact equation and find the interval of validity for the solution

i) $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0 \quad y(0) = -3$

Sol

$M = 2xy - 9x^2$

$N = 2y + x^2 \qquad M_y = 2x$

$\qquad\qquad\qquad N_x = 2x$

Now, how do we actually find $\psi(x, y)$?

$\psi_x = M$

$\psi_y = N$

$\psi = \int M dx$ or $\psi = \int N dy$

$\psi_y = x^2 + h'(y) = 2y + x^2 + 1 = N$

$h'(y) = 2y + 1$

$h(y) = \int 2y + dy + y^2 + y + k$

$\psi(x, y) = x^2 y - 3x^2 + y^2 + k = y^2 + (x^2 + 1)y - 3y + k$

$y^2 + (x^2 + 1)y + 3x^2 + k = c$

$y^2 + (x^2 + 1)y - 3x^3 = c$

initial condition to find c

$(-3)^2 + (0 + 1)(-3) - 3(0)^2 = c \Rightarrow c = 6$

$y^2 + (x^2 + 1)y - 3x^2 - 6 = 0$

Quadratic formula

$$y(x) = \frac{-(x^2+1) \pm \sqrt{(x^2+1)^2 - 4(1)(-3x - 6)}}{2(1)}$$

$$\frac{-(x^2+1) \pm \sqrt{x^4 + 2x^3 + 2x^2 + 25}}{2}$$

$$-3 = y(0) = \frac{-1 \pm \sqrt{25}}{2} = \frac{-1 \pm 5}{2} \quad \mp -3 \cdot 2$$

$$y(x) = \frac{-x^2 + 1 - \sqrt{x^4 + 2x^3 + 2}}{2}$$

$x^4 + 9x^2 + 25 = 0$

ii $\frac{2ty}{t^2+1} - 2t - (2 - \ln(t^2+1))\,y' = 0 \quad y(5) = 0$

Sol

$M = \frac{2ty}{t^2+1} - 2t \qquad M_y = \frac{2t}{t^2+1}$

$N = \ln(t^2+1) - 2 \qquad N_t = \frac{2t}{t^2+1}$

integrate the first one

$\psi(x,y) = \int \frac{2ty}{t^2+1} - 2t \, dy = y\ln(t^2+1) - t^2 + h(y)$

Now differentiate

$\psi_y = \ln(t^2+1) + h'(y) = \ln(t^2+1) - 2 = h'(y) = -2 \quad h(y) = -2y$

$\psi(t,y) = y\ln(t^2+1) - t^2 - 2y$

$y\ln(t^2+1) - t^2 - 2y = c$

$c = -25$

$\ln(t^2+1) - 2 = 0$

$\ln(t^2+1) = 2$

$t^2+1 = e^2$

$t = \pm\sqrt{e^2-1}$ As

THE END