Linear Algebra **name junaid ur rehman** **id 11484 (BEE)**
BS-SE 2nd Semester

Pray for the safety of humanity.
Pray for all the Muslims and Pakistanis wherever they are.
Pray for our university, staff and students.

Instructions:

This is an open-book take-home mid-term assignment, to be submitted till Monday, April 26th, 2020. You may consult the textbook, your notes, and any material posted on SIC. No other sources of information are allowed, including friends, classmates, materials from other classes, tutors, etc. Please write your solutions as clearly and neatly as possible. Also, show all your work, preferably with explanations for each step. If you are asked to do a problem a specific way (for example, "use the standard matrix representation. . . "), then you will receive no credit for doing it any other way. You will also receive no credit for answers without sufficient work to produce them. Attempt all questions. Answers copied will both be marked zero. Late submission will not be accepted and marked zero.

<u>How to Submit?</u>

1. ***Write your names and Ids at the top of answer sheet.***
2. ***Scan / Take Photo of each paper and save each photo with a number. E.g. photo of paper 1 of answer sheet be saved with name 1.jpg, then 2.jpg and so on.***
3. ***Put all answer photos in a word file by simply copy and pasting images, name the document with subject name, your name and id e.g. LA_Ali_12345.***
4. ***You will be provided upload link on sic to submit your answers.Go to Lectures section and click on Upload Assignment and upload your answers document file in the subject.Different formats are mentioned for uploading assignment, student can choose any one of them.***
5. ***Due date and remaining time will be shown on the same window.***

Q. No. 1 Consider the given below matrix as the augmented matrix of a linear system. Explain in your words the next elementary row operation that should be performed in order to solve this system. Where ID3 is the 3rd digit in your ID and ID_last is the last digit of your ID in inverse e.g. if your ID is 12345 then –ID_last = -5.

$$\begin{bmatrix} 1 & ID3 & 3 & 0 & 5 \\ 0 & 1 & -ID\_Last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID3 \end{bmatrix}$$

Q. No. 2 (a) Find the elementary row operation that transforms the first matrix into second and reverse row operation that transforms the second matrix into first

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

(b) Given below are some matrices. Find whether these are in the forms written in front of them or not. Explain in your own words for each of the selection in detail.

a. $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \Pi & 0 & 0 \\ 0 & 0 & -\Pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is in echelon form

b. $\begin{bmatrix} 1 & 0 & \Pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form

c. $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

d. $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form

Q. No. 3

(a) The row echelon form is used to solve the system of linear equations. What is the difference between the row echelon and reduced row echelon form? What is the practical use of reduced row echelon form? Give one example.

(b) Find an echelon form for the below matrix using row operations. Where

ID2 is 2nd digit in your ID e.g. if your ID is 12345 ID2 = 2, ID3=3, ID_first_last is the first and last digit of your ID i.e.15

$$\begin{bmatrix} 1 & \mathrm{ID2} & 8 \\ 2 & 8 & -1 \\ -\mathrm{ID3} & 0 & 0 \\ 1 & -4 & \mathrm{ID\_First\_Last} \end{bmatrix}$$

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Q1 # Consider the given below matrix as the augmented matrix of a linear system. Explain in your own words the next elementary row operation that should be performed in order to solve this system. Where ID3 is the 3rd digit in your ID. and ID last is the last digit of your ID in inverse e.g if your ID is 12345 then -ID - last = -5?

$$\begin{bmatrix} 1 & ID3 & 3 & 0 & 5 \\ 0 & 1 & -ID\text{-}last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID3 \end{bmatrix}$$

Sol #

$$\begin{bmatrix} 1 & 4 & 3 & 0 & 5 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

let the matrix is A

$$x + 4y + 3z + 0j = 5$$
$$0x + y - 4z + 0j = 7$$
$$0x + 0y + z + 0j = -6$$
$$0x + 0y + 0z + 1j = 4$$

(2)

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$$\begin{bmatrix} 1 & 4 & 3 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ j \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ -6 \\ 4 \end{bmatrix}$$

let the Augmented matrix be

$$\left[\begin{array}{cccc|c} 1 & 4 & 3 & 0 & 5 \\ 0 & 1 & -4 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$$= \begin{bmatrix} 1 & 0 & 19 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -23 \\ -17 \\ -6 \\ 4 \end{bmatrix} \begin{matrix} R_1 - 4R_2 \\ R_2 + 4R_3 \\ \\ \\ \end{matrix}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 91 \\ 0 & 1 & 0 & 0 & -17 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \begin{matrix} R_1 - 19R_3 \\ \\ \\ \\ \end{matrix}$$

$x = 91$

$y = -17$

$z = -6$

$j = 4$

(5)

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Q2(a) :→ Find the elementary row operation that transforms the first matrix into second and reverse row operation that transforms the second matrix into first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Sol#:

let first matrix be A,

let Second matrix be B

Elementary Row operation:

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$$R_3 \to R_3 - 2R_2$$

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Reverse Row operation:

$$B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$R_3 \to 2R_2 + R_3$$

$$B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$ Hence proved.

(4)

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Q2(b) :- Given below are some matrices. Find whether these are in the form written in front of them or not. Explain in your own words for each of the selection in detail?

(a) $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is in echelon form.

(b) $\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form

(c) $\begin{vmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{vmatrix}$ is in reduced row echelon form

(d) $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form.

Sol#

Let, (a) $\Rightarrow A = \begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is in echelon form

Yes, matrix A is in echelon form because of its definition as echelon form of a matrix state that "If a column contains a leading entry then are zero.

In matrix A, it satisfies the definition of echelon form of a matrix, so it is in echolen form.

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(b) $\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form. (5)

Sol#

Let $B = \begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Yes matrix B is in echelon form because of its definition, which states that "If a column contains a leading entry then all entries below that leading entry are zero.

matrix B column contain leading entries as 1 and below that all entries are zero so matrix B is in echelon form.

(C) :
$$\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$ is in reduced row echelon form

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Solution #

Let $C = \begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

$C = \begin{bmatrix} 5/5 & 0/5 & 0/5 & 7/5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

$R1/5$

$C = \begin{bmatrix} 1 & 0 & 0 & 7/5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

= Yes matrix C is in reduced row echelon form. because reduce row echelon form states that 'In reduced row echelon form the leading co-efficient must be 1. in each row is to the right of the leading co-efficent in the row above it. According to definition matrix C satisfies the definition properties so it is in reduced row echelon form.

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(d) $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form.

Sol:

let $D = \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

No matrix D is not in reduced row echelon form because if a matrix is in reduced row echelon form then it rows (non-zero) contains. it's firs entries as a number 1. matrix D the zero row is located in mid of matrix ~~so thae ze~~. So it is not in reduced row echelon form.

Q3(a):- The row echelon form is used to solve the system of linear equations what is difference between the row echelon and reduced row echelon form? What is the practical use of reduced row echelon form? Give one example?

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Ans# <u>Difference b/w row echelon and reduced row echelon form:-</u>

Row Echelon form	Reduced row Echelon form.
(1) Row echelon form of a matrix is define as " the leading entry in row echelon form in each row (Column) is the only non-zero entry in its row (Column).	(1) Reduced row echelon form is defined as: In reduced row echelon form the leftmost non-zero entry of a row is equal to 1. The leftmost non-zero entry of a row is the only non zero entry in its column.
2) Echelon form of a matrix is not unique which mean there are infinite answers possible when we perform row reduction or elementry operation.	(2) Reduced row echelon form is unique. Which means when we apply elementry row operation on a matrix it will produce the same answers. no matter how we perform the same row operations

(3) The entries only below the first leading non-zero entry that must be zero not necessary for above one's

(4) Each row containing a non-zero number 1 appearing in the row's first non zero column. Such entry will be known as "leading entry one"

Example

$$\begin{vmatrix} 1 & 0 & 2 & 3 & 4 \\ 0 & 1 & 5 & 6 & 7 \\ 0 & 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

If a column contain a leading entry then all entries below that leading entry are zero.

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(3) The entries above and below the first 1 in each row must all be zero.

(4) In reduced row echelon form, the left most non-zero entry of a row is equal to 1. The leftmost non-zero entry of a row is the only non-zero entry in its column.

Example

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

main diagonal entries are 1
All other entries are 0

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Q 3(b) :- Find echelon form for the below matrix using row operation. where ID2 is 2nd digit in your ID e.g if your ID is 12345 ID2 = 2 , ID3 = 3. ID - first - last is the first and last digit of your ID i.e 15

$$\begin{bmatrix} 1 & ID2 & 8 \\ 2 & 8 & -1 \\ -ID3 & 0 & 0 \\ 1 & -4 & ID\text{-first}.\text{last} \end{bmatrix}$$

Sol #

$$A = \begin{bmatrix} 1 & 1 & 8 \\ 2 & 8 & -1 \\ -4 & 0 & 0 \\ 1 & -4 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 8 \\ 2 & 8 & -1 \\ 1 & -4 & 14 \\ -4 & 0 & 0 \end{bmatrix} R3 \longleftrightarrow R4$$

$$\begin{bmatrix} 1 & 1 & 8 \\ 0 & 16 & -29 \\ 0 & -16 & 336 \\ -4 & 0 & 0 \end{bmatrix} \begin{matrix} R_2 - 2R_3 \\ 4R_3 + R_4 \end{matrix}$$

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$$= \begin{bmatrix} 1 & 1 & 8 \\ 0 & 16 & -29 \\ 0 & 0 & 167 \\ -4 & 0 & 0 \end{bmatrix} R_3 + R_2$$

$$= \begin{bmatrix} 1 & 1 & 8 \\ 0 & 16 & -29 \\ 0 & 0 & 167 \\ 0 & 4 & 32 \end{bmatrix} R_4 + 4R_1$$

$$= \begin{bmatrix} 1 & 1 & 8 \\ 0 & 16 & -29 \\ 0 & 0 & 167 \\ 0 & 0 & 157 \end{bmatrix} 4R_4 - R_2$$

$$= \begin{bmatrix} 1 & 1 & 8 \\ 0 & 1 & -29/16 \\ 0 & 0 & 167 \\ 0 & 0 & 157 \end{bmatrix} \frac{1}{16} \times R_2'$$

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$$= \begin{bmatrix} 1 & 1 & 8 \\ 0 & 1 & -29/16 \\ 0 & 0 & 1 \\ 0 & 0 & 157 \end{bmatrix} \frac{1}{167} \times R_3$$

$$= \begin{bmatrix} 1 & 1 & 8 \\ 0 & 1 & -29/16 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} R_4 - 157\, R_3$$

Answer.