SHAH HASSAN

ID - 7978

SEC — B

SUB — Numerical Analysis

TO- Ma'am SHOMAILA MAZHAR

Q No - 01

$$\frac{dy}{dx} = 2x \; ; \; y(0) = 1$$

$$h = 0.1$$

Solution:

$$f(x,y) = 2x$$

$$x_0 = 0, \; y_0 = 1$$

$$h = 0.1$$

$$x_{n+1} = x_n + h$$

Put, $n = 0$

$$x_1 = x_0 + h$$
$$= 0 + 0.1$$
$$= 0.1$$

$$x_2 = x_1 + h$$
$$= 0.1 + 0.1$$
$$= 0.2$$

$$x_3 = 0.3$$

$$x_4 = 0.4$$

$$x_5 = 0.5$$

1st Integration:

Euler's formula

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$n = 0$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + 0.1\,[(0)(1)]$$

$$\boxed{y_1 = 1.1}$$

Modefied Eulers formula

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right]$$

$$y_1 = y_0 + \frac{h}{2}\left[(x_0, y_0) + (x_1, y_1)\right]$$

$$y_1 = 1 + \frac{0.1}{2}\left[(0)(1) + (0.1)^2(1.1)\right]$$

$$y_1 = 1 + 0.05\,(2.2)$$

$$\boxed{y_1 = 1.11}$$

2nd Integration:

Euler's formula

$$y_2 = y_1 + hf(x_1, y_1)$$

$$y_2 = 1.11 + 0.1\left[(0.1)^2(1.11)\right]$$

$$y_2 = 1.11 + 0.1(1.21)$$

$$\boxed{y_2 = 1.231}$$

Modified Euler's formula

$$y_2 = y_1 + \frac{0.1}{2}(x_1, y_1) + (x_2, y_2)$$

$$y_2 = 1.11 + 0.05\left[(0.1)^+(1.11) + (0.2)^+(1.231)\right]$$

$$y_2 = 1.11 + 0.05\left[1.21 + 1.431\right]$$

$$\boxed{y_2 = 1.242}$$

3rd Integration:

Euler's formula

$$y_3 = y_2 + hf(x_2, y_2)$$

$$y_3 = 1.242 + 0.1(0.2)^+(1.242)$$

$$y_3 = 1.242 + 0.1442$$

$$\boxed{y_3 = 1.3862}$$

Modefied Eulers formula

$y_3 = y_2 + 0.1/2\ [x_2 + y_2] + [x_3 + y_3]$

$y_3 = 1.242 + 0.05\ [(0.2) + (1.242) + (0.3) + (1.3862)]$

$y_3 = 1.242 + 0.05\ [1.442 + 1.6862]$

$\boxed{y_3 = 1.398}$

Q No 02

Use the fourth Runge Kutta Method to obtain a solution of

$$\frac{dy}{dx} = x^2 + x - y$$

Subject to $y=0$ when $x=0$, for $0<x\le 0.6$ with $h=0.2$ work through out to four decimal places.

Solution:

Given.

$y=0,\ x=0,\ h=0.2 \quad 0\le x\le 0.6$

$y_{n+1} = y_n + K$

1st iteration:

$n=0$

$y_1 = y_0 + K,\ K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$K_1 = hf(x_n, y_n)$

$K_1 = h(x_0^2 - x_0 - y_0)$

$K_1 = 0.2(0^2 - 0 - 0)$

$$\boxed{K_1 = 0}$$

$$K_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}\right)$$
$$= 0.2 f\left(x_0 + h/_2, y_0 + h/_2\right)$$
$$= 0.2 f\left(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2}\right)$$
$$= 0.2 f(0.1, 0.1)$$
$$= 0.2 \left(0.1^2 + \cancel{0.1} - \cancel{0.1}\right)$$

$$\boxed{K_2 = 0.0020}$$

$$K_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{K_2}{2}\right)$$
$$= 0.2 f\left(0 + \frac{0.2}{2}, 0 + \frac{0.002}{2}\right)$$
$$= 0.2 f(0.1, 0.001)$$
$$= 0.2 \left(0.1^2 + 0.1 - 0.001\right)$$

$$\boxed{K_3 = 0.0218}$$

$$K_4 = hf\left(x_n + h, y_n + K_3\right)$$
$$= 0.2 f(0 + 0.2, 0 + 0.0218)$$
$$= 0.2 f(0.2, 0.0218)$$
$$= 0.2 \left(0.2^2 + 0.2 - 0.0218\right)$$

$$\boxed{K_4 = 0.0436}$$

$$K = \frac{1}{6}\left(0 + 2(0.002) + 2(0.0218) + 0.0436\right)$$

$$K = 0.0152$$

$$y_1 = 0 + 0.0152$$

$$\boxed{y_1 = 0.0152}$$

Q No 03

Given data:

$$a = 0,\ b = 10,\ n = 10$$

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

Solution:

x	0	1	2	3	4	5	6	7	8	9	10
$f(x_0)$	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

Using formula

$$f(x)dx = \frac{h}{2}\left[f(x_0) + 2f(x_1) + f(x_2) + f(x_3) \cdots f(x_9) + f(x_{10})\right]$$

$$= \frac{1}{2}\left[10.1 + 2(17.2 + 24.4 + 29.2 + 34.6 + 41.2\right.$$

$+ 50.9 + 57.8 + 60.3 + 61.2) + 62.1]$

$\boxed{= 412.9 \text{ Ans}}$

Q No-04

Estimate the values of the following integrals using Simpson's Rule.

$\int_2^3 \ln(x^3+1)\, dx$. Use 10 strips.

Solution:

$n = 10$

$h = \frac{3-2}{10} = 0.1$

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$f(x)$	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777	1.922	2.062

Now using formula

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4f(x_1 + f(x_3) \ldots\ldots\right] + 2$$

$$\left[f(x_2 \ldots) + f(x_n)\right]$$

$$= \frac{0.1}{3}\left[0.693 + 4(0.846 + 1.162 + 1.476 + 1.777) + \right.$$

$$2(1.003 + 1.320 + 1.628 + 1.922) + 2.062$$

$\boxed{= 1.184}$ Ans.