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Date 24/9/20

Note: Attempt all questions

Q NO# 1

part (a)

$$= \frac{3x^4 - 2x^3 + 5}{x^3 + 1}$$

- Applying quotient rule

$$= \frac{d}{dx}\frac{3x^4 - 2x^3 + 5}{x^3+1} = \frac{d}{dx}(3x^4 - 2x^3 + 5)(x^3+1) - 3x^4 - 2x^3 + 5\,\frac{d}{dx}\,x^3+1$$

$$= (12x^3 - 6x^2)(x^3+1) - (3x^4 - 2x^3 + 5)(3x^2)$$

$$= 12x^6 - 6x^5 + 12x^3 - 6x^2 - (9x^6 - 6x^5 + 15x^2)$$

$$= 12x^6 - 6x^5 + 12x^3 - 6x^2 - 9x^6 + 6x^5 - 15x^2$$

$$= 3x^6 + 12x^3 - 21x^2$$

$3x^6 + 12x^3 - 21x^2$ Ans

Part (b)

$$= \frac{(x^3+1)^2}{x^3-1}$$

$$= \frac{x^6+1+2x^3}{x^3-1}$$

Applying quotient rule

$$= \frac{x^6+2x^3+1}{x^3-1} = \frac{d}{dx}(x^6+2x^3+1)(x^3-1) - d/dx\,(x^3-1)(x^6+2x^3+1)$$

$$= (6x^5+6x^2)(x^3-1) - (3x^2)(x^6+2x^3+1)$$

$$= 6x^8 + 6x^5 - 6x^5 - 6x^2 - (3x^8+6x^5+3x^2)$$

$$= 6x^8 - 6x^2 - 3x^8 - 6x^5 - 3x^2$$

$$= 3x^8 - 6x^5 - 9x^2 \quad \text{Ans}$$

Q NO#2 (3)

part (a)

$$\int \frac{1}{\sqrt{x^5}}\,dx \Rightarrow \int x^{-5/2}\,dx$$

$$\Rightarrow \frac{x^{-5/2+1}}{-5/2+1}$$

$$= \frac{x^{-3/2}}{-3/2} + C$$

Part (b) ④

$$\int \frac{1}{(8x+7)^8}\, dx$$

let $u = 8x+7$

$$\frac{du}{dx} = 8x+7$$

$$\frac{du}{dx} = 8$$

$$dx = 1/8\, du$$

$$\int \frac{1}{(u)^8} \frac{1}{8} du$$

$$\frac{1}{8}\left[\frac{u^{-8+1}}{-8+1} + C\right]$$

$$\frac{1}{8} \frac{u^{-7}}{-7} + C$$

$$\frac{1}{-56}\left[u^{-7} + C\right]$$

putting values

$$\frac{1}{-56}\left[(8x+7)^{-7} + C\right]$$

Q NO#3

Part (a)

$$\int \frac{-x+9}{2x^2-8x+6}$$

$$\int \frac{-x+9}{2x^2-6x-2x+12} \Rightarrow \int \frac{-x+9}{2x(x-6)-2(x-6)}$$

$$\Rightarrow \int \frac{(-x+9)}{(2x-2)(x-6)} - \frac{A}{2x-2} + \frac{B}{x-6}$$

multiply $(2x-2)(x-6)$ on both sides

$$-x+9 = A(x-6) + B(2x-2) \quad \text{—(A)}$$

putting $x=1$ in eq (A)

$$-1+9 = A(1-6) + B(2(1)-2)$$

$$8 = A(-5) + B(2-2)$$

$$8 = A(-5)$$

$$A = -8/5$$

putting $x=6$ in eq (A).

$$-6+9 = A(6-6) + B(2(6)-2)$$

$$3 = B(10)$$

$$\boxed{B = 3/10}$$

⑥

$$= \int \frac{-x+9}{(2x-2)(x-6)} = \frac{-8/5}{2x-2} + \frac{3/10}{x-6}$$

$$= -5/8 \int \frac{1}{2x-2} dx + \frac{10}{3} \int \frac{1}{x-6}$$

$$-5/8 \ln|2x-2| + \frac{10}{3} \ln|x-6|$$

Part (b)

$$\int \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)}\,dx$$

let

$$= \int \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2x+3}$$

= multiply $(x^2+1)(x^2+2x+3)$ on both sides.

$$= 4x^2+8x = Ax+B(x^2+2x+3)+Cx+D(x^2+1)$$

$$= 4x^2+8x = Ax^3+2Ax^2+3Ax+Bx^2+2Bx+3B+Cx^3+Cx+Cx+Dx^2+D$$

Equating co-effecient of x^3, x^2, x & 1

$x^3 \Rightarrow 0 = A+C$ — (1) $\Rightarrow A = -C$ — (A)

$x^2 \Rightarrow 4 = 2A+B+D$ — (2)

$x \Rightarrow 8 = 3A+2B+C$ — (3)

$1 \Rightarrow 0 = 3B+D$ — (4) $D = 3B \rightarrow$ (B)

putting the value of & D in eq (2) & (3)

$A = C$, $D = 3B$

eq (2)

$\Rightarrow 4 = -2C+B+3B$

$\Rightarrow 4 = -2C+4B$ — (C)

eq (3)

$\Rightarrow$

$8 = -3C + 2B + C$

$8 = -2C + 2B$ — (D)

Sub eq C & (D)

$4 = -2C + 4B$

$\pm 8 = \overset{-}{+}2C \overset{+}{-}2B$

$-4 = 2B$

$\boxed{B = -2}$

putting the value of B in eq (C)

$4 = -2C + 4(-2)$

$4 = -2C - 8$

$4 + 8 = -2C$

$\boxed{C = -6}$

putting the value of C in eq (A)

$\boxed{A = 6}$

putting value of B in eq (D)

$D = 3(-2)$

$D = -6$

⑨

$$\int \frac{4x^2+8x}{(x^2+1)(x^2+2x+3)}\,dx = \int \frac{6x+(-2)\,dx}{x^2+1} \qquad \frac{-6x+-6}{x^2+2x+3}$$

$$\int \frac{6x-2}{x^2+1} - \int \frac{(2x+2)^3}{x^2+2x+3}$$

$$3/6 \int \frac{2x}{x^2+1} - 2\int \frac{1}{x^2+1} - \int \frac{2x+2}{x^2+2x+3} \int \frac{3}{x^2+2x+3}$$

$$1/2 \ln|x^2+1| - 1/3 \ln|x^2+1| - 1/6 \ln x^2+2x+3$$

$$\left[\frac{3}{x^2+2x+3}\right]$$

Q NO # 4 (20)

Part (a)

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & +2 \\ -5 & -1 \end{bmatrix}$$

Part (b)

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

(11)

Part (c)

$$X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

let $I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$X = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -3 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

Q NO#5 (12)

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$A^2 + BC = ?$

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+4\times2 & 1\times4+4\times1 \\ 2\times1+1\times2 & 2\times4+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+8 & 4+4 \\ 2+2 & 8+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix}$$

$BC = ?$

$$= \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3\times1+2\times0 & -3\times0+2\times2 \\ 4\times1+0\times0 & 4\times0+0\times2 \end{bmatrix}$$

$$= \begin{bmatrix} -3+0 & 0+4 \\ 4+0 & 0+0 \end{bmatrix}$$

$$BC = \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$A^2 + BC = ?$$

$$= \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9+(-3) & 8+4 \\ 4+4 & 9+0 \end{bmatrix}$$

$$= \begin{bmatrix} 9-3 & 8+4 \\ 4+4 & 9+0 \end{bmatrix}$$

$$= \begin{bmatrix} 9-3 & 12 \\ 8 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}$$