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Section A

Subject Advance fluid Mechanics

Submitted to Engr Abdul Waheed

Date:



# Q.No 1

## Part (a)

**Define drag with its components ...... laminar & turbulent boundary layers.**

**Ans:-**
A body which is wholy immersed in a homogenous fluid may be subjected two kind of Forces arising from relative motion b/w body & fluid. These forces are termed as drag & lift depending on forces parallel or right angle to motion.
Drag Force are submerged body can have 2 components.

**Pressure Drag:-** It is equal to integration of components in direction of motion of all pressure forces exerted on surface of body.

$F_P = C_P \cdot \rho \frac{V^2}{2} \cdot A$ (where $C_P$ depend on shape).

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## Friction drag:-

It is equal to integration of component of all shear stress along the surface in direction of motion.

$$F_D = C_f \cdot \frac{\rho V^2}{2} \cdot BL$$

($C_f$ depend on velocity)

## Friction Drag of Boundary layer:-

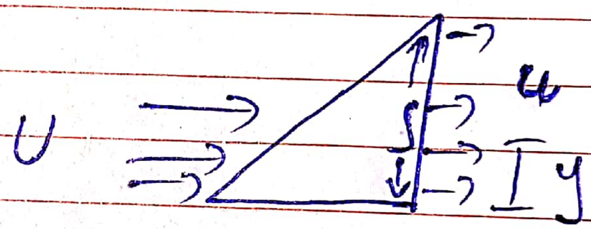
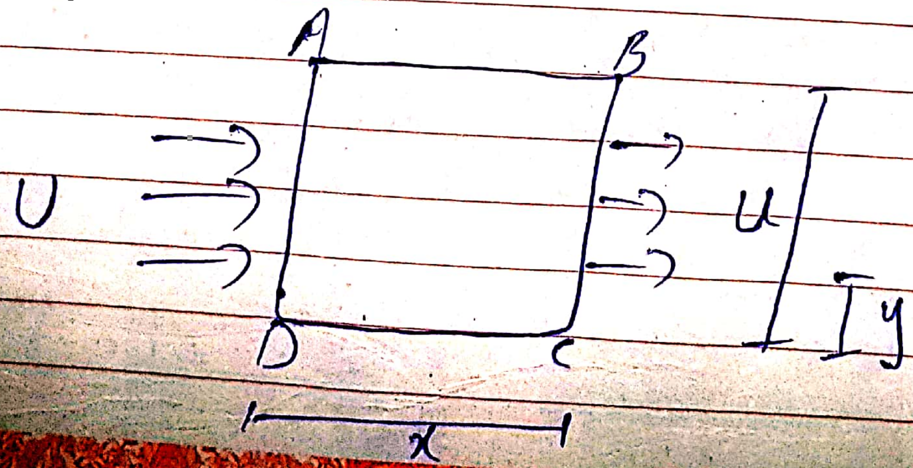



Figure shows growth of Boundary Layer along one side of smooth plate in steady flow of incompressible fluid consider value where is the thickness of boundary layer & U is undisturbed velocity.



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As we have $\Sigma F_x = 0$

where

$$F_u = \frac{\Delta P}{\Delta t} = \frac{\Delta m v}{\Delta t} \quad \because m = \rho v$$

$$F_u = \frac{\Delta \rho . Vol. V}{\Delta t} = \Delta \rho Q v$$

$$F_u = \Delta \rho Q v$$

$-F_u$ = rate of change of BC + AB − AD

$$AD = \rho U (U \delta B)$$

$$BC = \rho B \int u^2 dy$$

$$AB = \rho U (U B \delta) - B \int_0^\delta u dy$$

$$AB = \rho U (U B \delta) - B \int_0^\delta u dy$$

$$F_u = \rho B \int_0^\delta u (U - u) dy$$

integration on b/s → (1)

where a is a function of boundary layer velocity distribution

Now to find shear stress

$$\tau = \frac{F_u}{A} = \frac{dF_u}{B du} = \frac{dF_u}{B du}$$

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(4)

$$\tau_o = \frac{\rho U^2 \alpha \, d\delta}{\rho \, dx} = \rho U^2 \alpha \frac{d\delta}{dx}$$

$$\tau_o = \rho U^2 \alpha \frac{d\delta}{dx}$$

## Laminar boundary layer:-

$$\frac{u}{U} = f\left(\frac{y}{\delta}\right) \rightarrow (1)$$

$$\frac{y}{\delta} = \eta \Rightarrow y = \delta\eta$$

$$dy = \delta d\eta \rightarrow (2)$$

$$\frac{u}{U} = f(\eta)$$

$$du = U df(\eta) \rightarrow (3)$$

For laminar flow

$$\tau_o = \mu \frac{du}{dy} \rightarrow (4)$$

$$\tau_o = \frac{\mu U df(\eta)}{\delta d(\eta)}$$

$$\tau_o = \frac{\mu U \beta}{\delta} \rightarrow (5)$$

As we have $\tau_o = \rho u^2 \alpha \frac{d\delta}{du}$

Compare both

$$\rho u^2 \alpha \frac{d\delta}{dx} = \frac{\mu U B}{\delta}$$

$$\delta\, d\delta = \frac{\mu B\, dx}{\rho U \alpha}$$

Int on b.s

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} x + c \qquad \because c = 0$$

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}} \qquad \because R_x = \frac{U x \rho}{\mu}$$

$B = 1.63$, $\alpha = 0.135$

$$\delta = \frac{4.91 x}{\sqrt{R_x}} \longrightarrow 6$$

Where $(R_x)$ is local reynold no.

As we have

$$\tau_o = \frac{\mu U B}{\delta}$$

Put eq (6) in (5)

$$\tau_o = 0.332 \frac{\mu U}{u} \sqrt{R_x}$$

now

$$F_f = B \int_0^L \tau_o \, du$$

where $\tau_o = 0.332 \frac{\mu U}{u} \sqrt{R_x}$

$$R_x = \frac{u U \rho}{\mu}$$

then putting values

$$F_f = 0.664 \sqrt{\rho \mu L U^3}$$

As we have

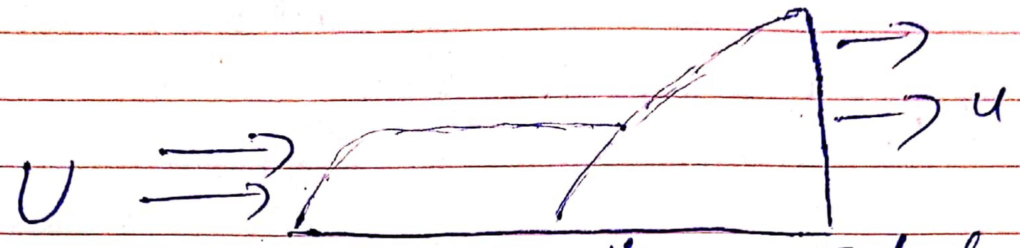
$$F_f = C_f \frac{\rho V^2}{2} B L$$

Thus equating Both side

$$C_f = 1.328 \sqrt{\frac{\mu}{\rho L U}} = \frac{1.328}{\sqrt{R_x}}$$

For laminar $R < 500{,}000$.

# Turbulent Boundary layer:-




The figure shows that velocity distribution of boundary layer which is steeper near walls & flater through out reminder of layer.

The shear stress is greater in turbulent than in laminar.

Thus $\tau_0 = \rho f \frac{V^2}{8}$

where V is the average velocity to obtain relation b/w average & max we have.

$$\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{f}} \quad \because f = 0.023$$

$$\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{0.023}}$$

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$U = 1.235 V$

$v = \frac{U}{1.235}$

& $F = \frac{0.316}{(Rn)^{1/4}}$ $\quad \because Rn = \left(\frac{DV}{\nu}\right)$

$\tau_0 = f \rho \frac{V^2}{8}$

$\tau_0 = \frac{0.316}{\left(\left(\frac{D}{\nu}\right)\left(\frac{U}{1.235}\right)\right)^{1/4}} \cdot \frac{\rho}{8}\left(\frac{U}{1.235}\right)^2$

$\tau_0 = \frac{0.023 \rho U^2}{\left(\frac{2\delta}{\nu}\right)^{1/4}} \rightarrow (1)$

As we have

$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx} \rightarrow (2)$

eq (1) & (2)

$u = 0 \quad , \delta = 0$

$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{\nu}{Uu}\right)^{1/5} \cdot x$

$\alpha = 0.0972$

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$$\delta = \frac{0.377}{(Rh)^{1/5}} \cdot x \longrightarrow 3$$

$$\tau_0 = 0.0587\, \rho \frac{V^2}{2} \left(\frac{\nu}{Vu}\right)^{1/5}$$

$$F_f = B \int_0^L \tau_0\, dx$$

$$F_f = 0.0735\, \rho \frac{U^2}{2} \left(\frac{\nu}{UL}\right)^{1/5} \cdot BL$$

$$F_f = c_f \cdot \rho \frac{V^2}{2} BL$$

equating b/s

$$C_f = \frac{0.0735}{(R)^{1/5}} \qquad (500000 < R < 10^7)$$

for $R > 10^7$

$$C_f = \frac{0.455}{(\log R)^{2.58}}$$

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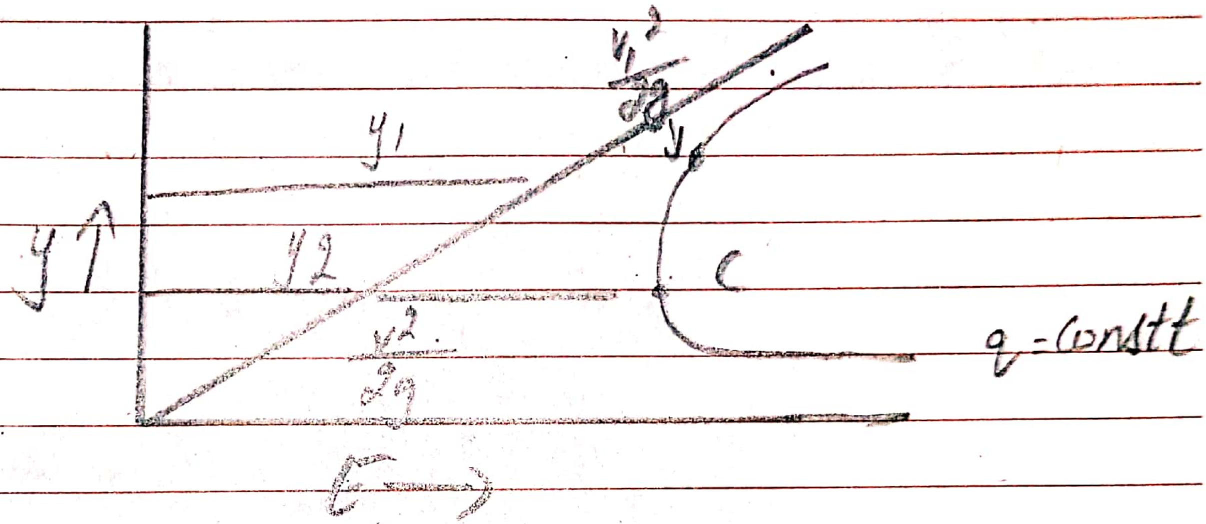
## Part (b)

Derive equation for critical ....- section of a channel.




This is specific energy eq:-

For particular $q$, there will be two kind of possible values of $y$ for given $E$. The eq is cubic with three roots with third being negative giving no values. Thus two alternative depths represents two totally different flow regims slow & deep on upper portion & Fast & shallow on lower portion. Point represent dividing point between two regima of flow.

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Thus for given 'q' value of E is minimum & Flow at this point is critical flow depth of flow at this point is critical depth i/c & velocity at this point is critical velocity.

Thus relation of critical depth can be found as

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

For minimum specific energy

$$\frac{dE}{dy} = 0$$

$$\frac{dE}{dy} = 1 - \frac{\cancel{2}}{\cancel{2}g}\left(\frac{q^2}{y^3}\right)$$

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3}$$

$$1 = \frac{q^2}{gy^3} = q^2 = gy^3$$

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

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As $q = Vy$ , $V_c^2 = g y_c$

OR $V_c = \sqrt{g y_c}$

$$y_c = \frac{V_c^2}{g}$$

Now $\frac{y_c}{2} = \frac{V_c^2}{2g}$

$$E_{min} = y_c + \frac{V_c^2}{2g} = y_c + \frac{y_c}{2}$$

$\frac{3}{2} y_c$ or $y_c = \frac{2}{3}$ constt

| | Sub critical | Critical | Super critical |
|---|---|---|---|
| Depth of Flow | $y > y_c$ | $y = y_c$ | $y < y_c$ |
| Velocity slop | $V < V_c$ mild slop $S_o < S_c$ | $V = V_c$ Critical slop | $V > V_c$ |

Scanned with CamScanner

# Q.2

Given :-

Depth of Rectangular channel (d) = ?

Flow rate (Q) = 3.5 m³/sec

Slope of Bed ($S_o$) = 0.0008

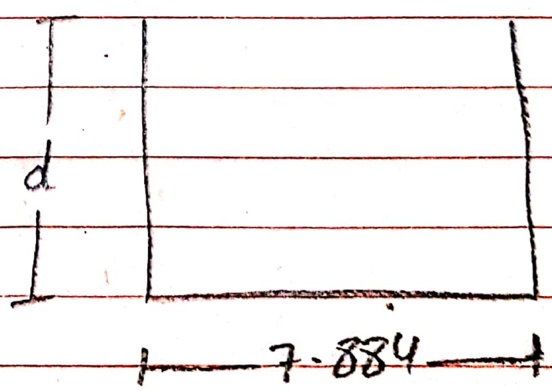
$n = 0.0219$

Width of Bed = 7884 mm
= 7.884

Critical depth = ?

Flow sub critical or super critical = ?

Sol:-




Area = 7.884 x d
= 7.884d

Perimeter = d + 7.884 + d
= 7.884 + 2d

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Hydraulic Radius $(R_h) = A/P$

$$= \frac{7.884d}{7.884+2d}$$

By Using Manning Equation

$$Q = \frac{1}{n} A . R_h^{2/3} . S_o^{1/2}$$

Putting values

$$3.5 = \frac{1}{0.0219} \times 7.884\,d \times \left(\frac{7.884d}{2d+7.884}\right)^{2/3} \times (0.008)^{1/2}$$

$$\boxed{d = 0.555m}$$

Area $= 7.884(0.555)$

$$\boxed{= 4.37m^2}$$

Perimeter $= 7.884 + 2(0.555)$

$$\boxed{= 8.994m}$$

Hydraulic Radius $(R_h) = \frac{4.37}{8.994}$

$$\boxed{= 0.485m}$$

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Finding critical depth:-

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

As $q = Q/B$

$$= \frac{3.5}{7.884}$$

$$= 0.443\ m^2/sec$$

$$\Rightarrow y_{cr} = \left(\frac{(0.443)^2}{9.81}\right)^{1/3}$$

$$= 0.271$$

As $y > y_{cr}$

$0.555 > 0.271$

So Flow is sub-critical.

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# Q. No 3

**Given data :-**

Wide of smooth plate = 200 mm

= 0.2 m

Length of smooth plate = 800 mm

= 0.8 m

Specific gravity = 0.89

Undisturbed velocity = 5 m/s

Kinematic viscosity = $0.93 \times 10^{-4}\ m^2/s$

**Required :-**

Friction drag = ?

**Sol:-**

$\nu = 0.93 \times 10^{-4}\ m^2/s$

At $x = L$ ,

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$$R = \frac{LU}{\nu}$$

$$= \frac{0.8 \times 5}{0.93 \times 10^{-4}}$$

$$= 43010.75$$

less than $R < 500{,}000$

Now $C_f = \frac{1.328}{\sqrt{R}}$

$$= \frac{1.328}{\sqrt{43010.75}}$$

$$= 6.40 \times 10^{-3}$$

$$= 0.0064$$

$$F_f = C_f \rho \frac{U^2}{2} BL$$

$$= 0.0064 \times 0.89 \times 1000 \times \frac{(5)^2}{2} \times 0.2 \times 0.8$$

$$\boxed{F_f = 11.39\ N}$$

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To find thickness of boundary layer

$$\frac{\delta}{x} = \frac{4.91}{\sqrt{143010.75}}$$

$$= 0.023$$

$\delta = 0.023 \times 0.8 \rightarrow$ full length

$$= 0.0184m$$

$$F_f = 0.664 B \sqrt{\rho \mu L U^3}$$

$$= 0.664 B \sqrt{\rho (\rho \times \nu) L U^3}$$

$$= 0.664 \times 0.2 \sqrt{0.89 \times 1000 \times (0.89 \times 1000 \times 0.93 \times 10^{-4}) \times 0.8 \times (5)^3}$$

$$\boxed{= 11.39N}$$