Department of Electrical Engineering
Assignment
Date: 14-04-2020

Course Details

Course Title: **Electro Magnetic Field Theory** **Module:** ____________

Instructor: Sir rafiq mansoor **Total Marks:** **30** .

Student Details

Name: **Talha Khan** **Student ID:** **13845**

Q1: Solve the following short Question	(a)	Transform the vector $B = yi\ (x+z)j$ located at point (-2,6,3) into cylindrical coordinates	Marks 2 CLO 1
	(b)	Convert the point (3,4,5) from Cartesian to spherical coordinates	Marks 2 CLO 1
	(c)	Find the spherical coordinates of A(2,3,-1)	Marks 2 CLO 1
	(d)	Find the Cartesian coordinates of B(4.25,120)	Marks 2 CLO 1
	(e)	Find the force between two charges when they are brought in contact and separated by 4cm apart, charges are 2nC and -1nC, in µN.	Marks 2 CLO 2
	(f)	Find the electric field intensity of two Charges -2C and -1C separated by a distance 1m in air	Marks 2 CLO 2
	(g)	Determine the charge that produce an 10^{-8} electric field strength of 40 v/cm at a distance of 30cm in vacuum (in c)	Marks 2 CLO 2
	(h)	A charge of $2 * 10^{-7}$ C is acted upon by a force of 0.1N. determine the distance to the other charge $4.5 * 10^{-7}$ of C, both the charges are in vacuum	Marks 2 CLO 2
Q2:	**(a)**	**Find the angle between the vectors shown in figure.**	**Marks 4**

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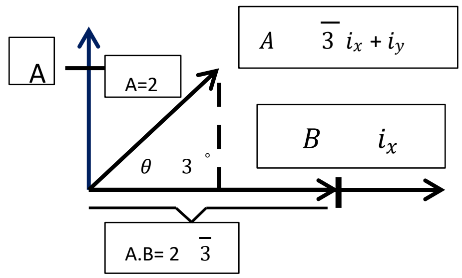
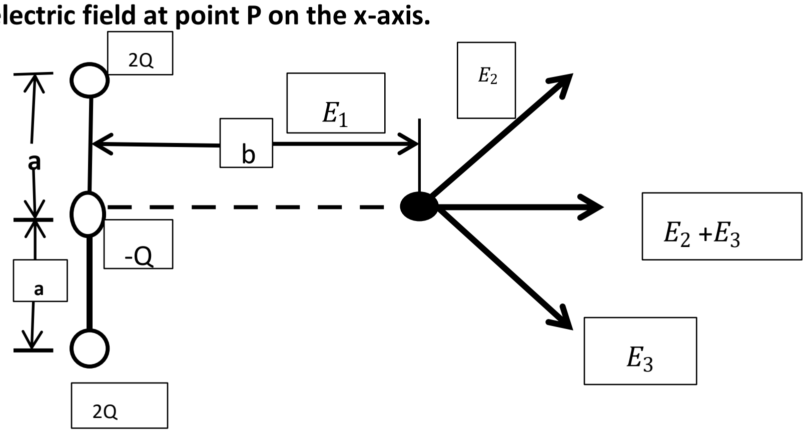
<tr><td></td><td></td><td>
</td><td>CLO 1</td></tr>
<tr><td></td><td>(b)</td><td>Find the gradient of each of the following functions where a and b are constant
(i) $f = ax^2 + by^3z$
(ii) $f = ar^2 \sin \emptyset + brz \cos 2 \emptyset$</td><td>Marks 4
CLO 1</td></tr>
<tr><td>Q3:</td><td></td><td>Three pointer charges are placed on the y-axis as shown. Find the electric field at point P on the x-axis.

</td><td>Marks 6
CLO 2</td></tr>
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Q1 (a) Transform the vector $B = yi(x+z)j$ located at point $(-2, 6, 3)$ into cylindrical coordinates.

Sol:- $B = yixj + yizj$

Point $x = -2$, $y = 6$, $z = 3$

From cartesian to cylindrical

$\rho = \sqrt{x^2 + y^2}$

$\rho = \sqrt{(-2)^2 + (6)^2} = \sqrt{4 + 36}$

$= \sqrt{40} = 6.324$

Now find the angle Φ

$\Phi = \tan^{-1}(y/x) = \tan^{-1}\left(\frac{6}{-2}\right)$

$\Phi = -71.56$

$Z = 3$

$B = [6.324 a_\rho - 71.56 a_\Phi + 3 a_z]$

(b) Convert the point $(3, 4, 5)$ from cartesian to spherical coordinates.

Sol:- For spherical we have to find r, θ, Φ

$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{3^2 + 4^2 + 5^2}$

(2)

$r = \sqrt{9+16+25} = \sqrt{50}$

$\boxed{r = 7.07}$

$\theta = \cos^{-1} \frac{z}{r} = \cos^{-1} \frac{5}{7.07}$

$\boxed{\theta = 45^\circ}$

$\phi = \tan^{-1} y/x = \tan^{-1} (4/3)$

$\phi = \boxed{53.13^\circ}$

$(r, \theta, \phi) = (7.07, 45^\circ, 53.13^\circ)$

(c) Find the ~~Cartesian~~ Spherical coordinates

$A (2, 3, -1)$

Sol :- $r = \sqrt{x^2+y^2+z^2} = \sqrt{(2)^2+(3)^2+(-1)^2}$

$r = \sqrt{4+9+1} = \sqrt{14}$

$\boxed{r = 3.74}$

$\theta = \cos^{-1} z/r = \cos^{-1} \frac{z}{\sqrt{x^2+y^2+z^2}}$

$\theta = \cos^{-1} \frac{-1}{3.74} = 105.5^\circ$

$\boxed{\theta = 105.5^\circ}$

$\phi = \tan^{-1}(y/x) = \tan^{-1}(3/2)$

$\boxed{\phi = 56.3^\circ}$

$(r = 3.74,\ \theta = 105.5^\circ,\ \phi = 56.3^\circ)$

(d) Find the Cartesian coordinates of
$B(4, 25, 120)$

<u>Sol</u>:- convert to cartesian Point B
is actually given in spherical
ie (r, θ, ϕ)

find (x, y, z) ?

find (x)

$x = r \sin\theta \cos\phi$

$x = 4 \sin 25 \cos 120$

$\boxed{x = -0.845}$

find (y)

$y = r \sin\theta \sin\phi = 4 \sin 25 \sin 120$

$\boxed{y = 1.463}$

(4)

find (z)

$z = r\cos\theta = 4\cos 25$

$\boxed{z = 3.625}$

$(x, y, z) = (-0.845, 1.463, 3.625)$

(e) $d = 4cm$, $q_1 = 2nC$, $q_2 = -1nC$.

Sol:- $F = K \frac{Q_1 Q_2}{r_2}$

where $K = \frac{1}{4\pi\varepsilon_0}$

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$$

$$F = \frac{2\times10^{-9} \times -1\times10^{-9}}{4\pi\times8.854\times10^{-12}\times(4\times10^{-2})^2}$$

$$F = -1.123\times10^{-5}$$

$\boxed{F = -11.23\,\mu N}$

(f) $q_1 = -2C$, $q_2 = -1C$

$d = 1m$ in Air $\Rightarrow r = 1m$

find $E = ?$

Sol:- due to Q_1

$$E_1 = \frac{q_1}{4\pi\varepsilon_0 R^2} = \frac{-2}{4\pi \times 8.85 \times 10^{-12} \times (1)^2}$$

$$E_1 = -1.798 \times 10^{10} \ N/C$$

due to Q_2

$$E_2 = \frac{q_2}{4\pi\varepsilon_0 R^2} = \frac{-1}{4\pi \times 8.854 \times 10^{-12} \times (1)^2}$$

$$E_2 = -8.9897 \times 10^{9} \ N/C$$

(g) $E = 40 \ V/cm$ $\quad E = 40 \times 100 \ V/m$

$E = 4000 \ V/m$

$r = 30 \ cm \Rightarrow r = 30 \times 10^{-2} m$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$Q = E \times 4\pi\varepsilon_0 r^2$$

$$Q = (40 \times 100)(4)(\pi)(8.854 \times 10^{-12})(30 \times 10^{-2} m)^2$$

$$\boxed{Q = 4 \times 10^{-8} C}$$

(h) $Q_1 = 2\times10^{-7}$, $Q_2 = 4.5\times10^{-7}$

$F = 0.2N$

distance ?

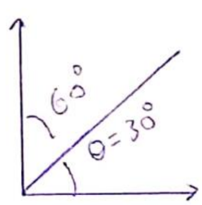
$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}$$

$$R^2 = \sqrt{\frac{Q_1 Q_2}{4\pi\varepsilon_0 F}} = \sqrt{\frac{2\times10^{-7}\times4.5\times10^{-7}}{4\times\pi\times8.854\times10^{-12}\times0.1}}$$

$$R^2 = \sqrt{8.088\times10^{-3}} \Rightarrow R = 8.088\times10^{-3}$$

$$\boxed{R = 0.0899\ m}$$

Q2: (a)



Sol:- $A = \sqrt{3}\, i_x + i_y$

$B = 2 i_x$

$A\cdot B = 2\sqrt{3}$

$A\cdot B = |A||B| \cos\theta_{AB}$

$|A| = 2$, $|B| = 2$

$2\sqrt{3} = |2||2| \cos\theta_{AB}$

$$\theta_{AB} = \cos^{-1}\left(\frac{2\sqrt{3}}{|2|\,|2|}\right)$$

$$\boxed{\theta_{AB} = 30^\circ}$$

Q.2(b) (i) $f = ax^2 + by^3 z$

Sol:- So, $f = ax^2 + by^3 z$

$$\nabla f = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k\right)(ax^2 + by^3 z)$$

$$\nabla f = \frac{\partial}{\partial x} ax^2 i + \frac{\partial}{\partial y} by^3 z j + \frac{\partial}{\partial z} by^3 z k$$

$$\nabla f = 2axi + 3bzy^2 j + by^3 k$$

$$\boxed{\nabla f = 2axi + 3bzy^2 j + by^3 k}$$

(ii) $f = ar^2 \sin\theta + brz\cos 2\phi$

Sol:- gradient in case of spherical

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{\partial \sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$$

$$\nabla f = \frac{\partial}{\partial r}(ar^2 \sin\phi + brz\cos 2\phi)\hat{r} + \frac{1}{r}\frac{\partial}{\partial \theta}$$
$$(ar^2 \sin\theta + brz\cos 2\phi)\hat{\theta} + \frac{1}{r\sin\theta}\cdot\frac{a}{\partial \phi}$$
$$(ar^2 \sin\phi + brz\cos 2\phi)\hat{\phi}$$

Then Now,

$$\nabla f = (2ar\sin\phi + bz\cos 2\phi)\hat{r} + \frac{1}{r}(0) + \frac{1}{r\sin\phi}$$
$$(ar^2\cos\phi - 2brz\sin\phi)\hat{\phi}$$

So,

$$\nabla f = (2ar\sin\phi + bz\cos 2\phi)\hat{r}$$
$$+ \frac{1}{r\sin\theta}\left(ar^2\cos\phi - 2brz\sin\phi\right)\hat{\phi}$$

Now In case of cylindrical

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial A_z}{\partial z}$$

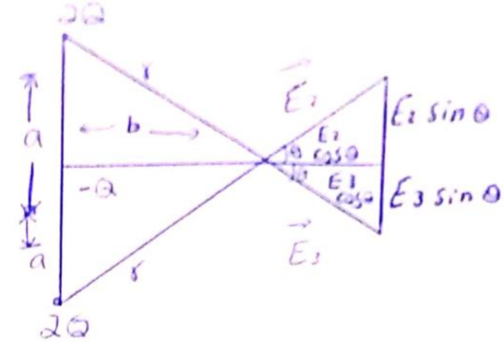
$$\nabla f = 0\hat{\rho} + \frac{1}{\rho}(ar^2\cos\phi - 2brz\sin 2\phi]\hat{\phi}$$
$$+ (br\cos 2\phi)\hat{z}$$

Then the first term is zero (0),

$$\boxed{\Delta f = \frac{1}{\rho}\left(ar^2\cos\phi - 2brz\sin 2\phi\right)\hat{\phi} + (br\cos 2\phi)\hat{z}}$$

Q(3)

Ans:- Sol:-



So $E_2 = E_3$ are both charges same

So, x-component becomes

$\vec{E}_{2+3} = 2\vec{E}\cos\theta \rightarrow$ eq (i)

Now, $E = K\frac{Q}{r^2}$

and (r) from Phythagorus theorm

$$r = \sqrt{a^2 + b^2}$$

$$E = \frac{KQ}{\sqrt{a^2+b^2}}$$

So eq (i) become

$$E_{2+3} = 2 \frac{KQ}{\sqrt{a^2+b^2}} \cos\theta$$

Now $E_1 = -\frac{KQ}{r^2}$

$$E_1 = -\frac{KQ}{b^2}$$

Total Electric field intensity at that point is :-

$$E = E_1 + E_{2+3}$$

$$= 2 \frac{KQ}{\sqrt{a^2+b^2}} \cos\theta - \frac{KQ}{b^2}$$

$$E = KQ\left(\frac{2}{\sqrt{a^2+b^2}} \cos\theta - \frac{1}{b^2} \right)$$

where $K = \frac{1}{4\pi\varepsilon_0}$