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SUBJECT :- DIFFERENTIAL EQUACTION.

Q1. Solve the following objectives types question

i - The order of matric A is MxP & the order of B is Pxn the order of Matrix AB is ?

Sol:- order of $A = m \times p$
order of $B = P \times n$

So order of $A \times B = M \times n$.

(ii) The number of non-zero rows in an Echelon form?

Sol:- The number of non zero rows is an Echelon form is called rank of the matrix.

For example.

$$A = \begin{vmatrix} 1 & 0 & -2 & 5 & 3 \\ -0 & 0 & 1 & -4 & 1 \\ -0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Since A is an row reduced form since it contains three non-zero rows, its row rank is Three.

(iii) If $B = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix}$ is a singular matric Then $a = ?$

Sol:- We know that for singular matric

$$|B| = 0$$

So

$$|B| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix}$$

$$= 1 \times a - 2 \times 4 = 0$$

$$a - 8 = 0$$

$$a = 8$$

(iv) If $A = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$ the $|A|$:-

Sol:- $A = \begin{vmatrix} 2i & -i \\ i & -i \end{vmatrix}$

Take modolus of a matrix A

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$|A| = 2i \times (-i) - i \times i$

$|A| = -2i^2 - i^2$

$|A| = -2(-1) - (-1)$

$|A| = 2+1$

$|A| = 3$ Ans.

v- The Matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Sol:- $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$

Here in, Matrix A diagnol element is same i.e. 9 so it is Scaler Matrix.

(vi) Solution of $dy/dx + 2xy = y = ?$

Sol:- $dy/dx = y - 2xy.$

$dy/dx = y(1-2x)dx$

$dy/y = (1-2x)\,dx$

$\int dy/y = \int (1-2x)dx$

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$\ln y = (x - x^2) + c_1$

$e^{\ln y} = e^{(x x^2) + c_1}$

$y = e^{x - x^2} e^{c_1}$

$y = ce^{x - x2}.$

(viii) The order & degree of differential equation $\frac{d^2 y}{dx^2} - 4xy = \sin\left(\frac{d2y}{dx^2}\right)$ is ?

2 But the degree is undefined because the unknown function y is an argument 1 transcendental sinfunction

(x) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Sol:- expand w.r.t column member first.

$1 \begin{bmatrix} b & b^2 \\ a & c^2 \end{bmatrix} - 1 \begin{bmatrix} a & a^2 \\ c & c^2 \end{bmatrix} + 1 \begin{bmatrix} a & a^2 \\ b & b^2 \end{bmatrix}$

$1(bc^2 - cb^2) - 1(ac^2 - ac) + i(a^2b - a^2b)$
$(bc^2 - cb^2) - (ac^2 - ac) + (ab^2 - a^2b)$

$bc(c-b) - ac(c-a) - ab(b-a)$

$bc(c-b) - ac(c-a) - ab(b-a)$
$bc^2 - cb^2 - ac^2 + a^2c + ab^2 - a^2b$
$(c-b)(bc - ac - ab - a^2).$

(vii) The order & degree of differential equati

$$(dy/dx)^3 = \sqrt{1+(dy/dx)^2}.$$

Sol:- Taking square on both sides.

$$\left(dy/dx)^3\right) = \left((1+(dy/dx)^2)^{1/2}\right)^2$$

$$(dy/dx)^6 = 1 + (dy/dx)^2$$

Degree $= 6$
order $= 1$.

~~Q. Express the d~~

(ix) The differential equation $2\frac{dy}{dx} + x^2y = 2x + 3, y(0) = 5$ i

Sol:- $2\frac{dy}{dx} + x^2y = 2x + 3$, $y(0) = 5$.

$\Rightarrow \frac{dy}{dx} + \frac{1}{2}x^2y = x + 3/2$ by (2)

$\Rightarrow \frac{dy}{dx} + \frac{1}{2}x(xy-1) = 3/2$

$\Rightarrow \frac{dx}{dx} = \frac{1}{2}x^2y + \frac{1}{2}2 + 3/2$

$$= y = \frac{1}{2}\frac{x^3}{3}y + \frac{1}{2}\ \frac{x^2}{2} + 3/2\,x + c_1$$

$$y = -\frac{1}{6}x^3 y + \frac{1}{4}x^2 + \frac{3}{2}x + c_1$$

at $x = 0,\ y = 5$

$$5 = 0 + 0 + 0 + 9$$

$$= 9 = 5$$

$$y = -\frac{1}{8}x^3 y + \frac{1}{4}x^2 + \frac{3}{2}x - 5$$

required Particular Solution.

10. Singular

Q.No.2:-

i. Express the determinant

$$\begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{bmatrix}$$

as the Product of factors which are linear in a, b, c.

ii. Find the Eigen Value. $\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

(i)

$$A = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} \quad \text{AdJ}(A) \quad \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$$

$$|A| = a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13} \quad - (1).$$

New.

$$A_{11} = (-1)^{1+1} M_{11} = (-1)^2 \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix}$$

$$A_{12} = (-1)^{1+2} M_{12} = (-1)^3 \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix}$$

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$$A_{12} = -\begin{vmatrix} a^2 & c \\ a^3 & c^3 \end{vmatrix}$$

$$A_{13} = (-1)^{1+3} M_{13} = (-1)^4 \begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

Put the Value in (1).

$$|A| = a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} + b(1)\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a\left[b^2c^3 - c^2b^3\right] - b\left(a^2c^3 - c^2a^3\right) + c\left(a^2b^3 - b^3c\right)$$

$$= ab^2c^3 - ac^2b^3 - ba^2c^3 + bc^2a^3 + ca^2b^3 - cb^2a.$$

$(x^2+3y^2)\,dy - 2xy\,dy = 0$ at $x = 2, y = 6.$

$$M dx + N dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} = 8\ 6y, \quad \frac{\partial N}{\partial x} = -2y.$$

$$\frac{My - Mx}{N} = \frac{6y-(-y)}{-2xy} = \frac{6y+2y}{-2xy} = \frac{\overset{4}{\cancel{8}}y}{-\cancel{2}xy}$$

$$= -\frac{4}{x}$$

$$I.F = e^{\int \frac{4}{x}dx} = e^{-4\int \frac{1}{x}dx} = e^{-4\ln n} = e^{\ln x^{-4}}$$

$$2\ x^{-4}$$

$$x^{-4}(x^2+3y^2)\,dx - \frac{2xy}{-x^4}\,dy = 6.$$

$$\left(\frac{1}{x^2}+\frac{3y^2}{x^4}\right)dx-\frac{2y}{x^3}dy=6$$

$$\frac{\partial M}{\partial y}=\frac{6y}{x^4},\quad \frac{\partial N}{\partial x}=+\frac{6y}{x^4}$$

$$\frac{\partial M_1}{\partial y}=\frac{\partial N_1}{\partial x}$$

General solut.

$$\int\frac{6y}{x^2}dx+\int 0\,dy=$$

$$6y\int\frac{1}{x^2}dx$$

$$-\frac{6y}{x}=C$$

$$-6y=Cx$$

No at $x=2,\ y=6$

$$-6(6)=C(2)$$
$$-12=2C$$
$$C=-6$$
$$-\frac{6y}{x}=-6$$
$$-6y=-6x$$
$$-6y+6x=0.$$

(ii)

let $A = \begin{vmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{vmatrix}$

let 't' is eigenvalue of A, Then

$$\det\left[A - t.I\right] = 0.$$

$$\det\left\{\begin{vmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{vmatrix} - \begin{vmatrix} t & 0 & 0 & 0 \\ 0 & t & 0 & 0 \\ 0 & 0 & t & 0 \\ 0 & 0 & 0 & t \end{vmatrix}\right\} = 0$$

$$\det\begin{vmatrix} 2-t & -1 & -1 & 0 \\ -1 & 3-1 & -1 & -1 \\ -1 & -1 & 3-t & -1 \\ 0 & -1 & -1 & 2-t \end{vmatrix} = 0$$

Expanding along R_1

$$\Rightarrow (2-t)\begin{vmatrix} 3-t & -1 & -1 \\ -1 & 3-t & -1 \\ -1 & -1 & 2-t \end{vmatrix} + 1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-t & -1 \\ 0 & -1 & 2-t \end{vmatrix} - 1\begin{vmatrix} -1 & 3t & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-t \end{vmatrix} = 0$$

$$\Rightarrow (2-t)\{(3-t)((3-t)(2-t)-1)+1(t-2-1)-1(1+3-t)\}-1(3-t(2-t)-1(t-2-0$$

$$-1(15-0)\}-1\{-1(t-2-1)-(3-1)(t-2-0)-1(1-0)\}=0.$$

$$\Rightarrow (2-t)\{(3-t)(6-3t-2t+t^2-1)+(t-3)-(4-t)\}+\{-1(6-3t-2t+t^2-1)+(t-2) -1\}$$

$$-1\{-1(t-3)-(3-t)(t-2)-1\}=0$$

$$\Rightarrow (2-t)\{(3-t)(t^2-5t+5)+(t-3)-4+t\}+\{-6+5t-t^2+1+t-2-1\}$$

$$-1\{3-t-(3t-6-t^2+2t)-1\}=0$$

$$\Rightarrow (2-t)\{3t^2-15t+15-t^3+5t^2-5t+2t-7\}+(-t^2+6t-8)-(3-t-3t+6+t^2-2t^3)(0)$$

$$\Rightarrow (2-t)(-t^3+8t^2-18t+8)-t^2+6t-8-t^2+6t+8=0$$

$$\Rightarrow 2t^3+16t^2-36t+\cancel{16}+t^4-8t^3+18t^2-8t-2t^2+12t-\cancel{16}=0$$

$$\Rightarrow t^4-10t^3+32t^2-32t=0.$$

$$\Rightarrow t(t^3-10t^2+32t-32)=0.$$

$$\Rightarrow t=0 \qquad t^3-10t^2+32t-32=0.$$

Now $t^3-10t^2+32t-32=0$

at $t=2$

$$L.H.S = 2^3-10(2)^2+32(2)-32$$

$$= 8-40+64+32 = 72-72=0$$

using synthetic division.

	1	-10	32	-32
2	↓	2	-16	32
	1	-8	16	0

$$\Rightarrow t^2-8t+16=0$$

$$\Rightarrow t^2-4t-4t+16=0.$$

$\Rightarrow t(t-4)(t-4)=0$

$\Rightarrow (t-4)(t-4)=0$

$t = 4, 4.$

So the required eigenvalues of Matrix A are $0, 2, 4, 4$.

QNO3:-
The rate of change in the form of different equation is given by

$(x^2+3y^2)dx - 2xydy = 0$ Find the general Solution at $= x=2$ & $y=6$.

Sol:- $(x^2+3y^2)\,dx - 2xydy = 0.$

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy} = \frac{1+3(y/x)^2}{2(y/x)}$$

its Homogenous.

Put $y = vx = \frac{y}{x} = v$

Diff w.r.t x

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{1+3v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+3v^2}{2v} - v$$

$$x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\frac{2v}{1+v^2}\,dv = \frac{dx}{x}$$

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{dx}{x}$$

$$\ln(1+v^2) = \ln x + \ln c$$

$$1+v^2 = cx$$

$$1+y^2/x = cx$$

$$\frac{x^2+y^2}{x^2} = cx$$

Now Put $x=2, y=6$

$$\frac{4+36}{4} = 2c$$

$$\frac{40}{4} = 2c$$

$$10 = 2c$$

$\boxed{C=5}$

Thus general Sol is given by

$$\frac{x^2+y^2}{x^2} = 5x$$

$\Rightarrow$ $x^2+y^2 = 5x^3$.