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Differential Equation

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Question - 1 $\quad 4y'' - 20y' + 25y = 0$

Solution:

The Auxilary equation

$$4\lambda - 20\lambda + 25 = 0$$

$$4\lambda^2 - 10\lambda - 10\lambda + 25 = 0$$

$$2\lambda(2\lambda - 5) - 5(2\lambda - 5) = 0$$

$$(2\lambda - 5)(2\lambda - 5) = 0$$

$$2\lambda - 5 = 0 \quad , \quad 2\lambda - 5 = 0$$

$$\lambda_1 = 5/2 \quad , \quad \lambda_2 = 5/2$$

$$\lambda_1 = \lambda_2 = 5/2$$

The roots are equal.

So,

$$y = (c_1 + c_2 x) e^{\lambda x} = (c_1 + c_2 x) e^{5/2 x}$$

—— x —— x ——

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Question - 2 (A)

$$y'' + 2y' + y = 0$$

$$y(0) = 4 \quad , \quad y(0) = -6$$

Solution -

Auxilary solution.

$$\lambda^2 + 2\lambda + 1 = 0$$

$$\lambda^2 + \lambda + \lambda + 1 = 0$$

$$\lambda(\lambda + 1) + 1(\lambda + 1) = 0$$

$$(\lambda + 1)(\lambda + 1) = 0$$

$$\lambda_1 = -1 \quad , \quad \lambda_2 = -1$$

Roots are equal & real.

$$y = (c_1 + c_2 x) e^{\lambda x}$$

$$y = (c_1 + c_2 x) e^{-1x}$$

Or

$$y = (c_1 e^{-x} + c_2 e^{-x})$$

where $x = 0$, $y = 4$

$$4 = c_1 e^0 + c_2 e^0$$

$$(4 = c_1)$$

Since,

$$y = c_1 e^{-x} + c_2 e^{-x}$$

$$y = -c_1 e^{-x} + c_2 e^{-x} - c_2 x e^{-x}$$

where,

$$x = 0, \qquad y = -6$$

$$-6 = -c_1 e^0 + c_2 e^0 - c_2 (0) e^0$$

$$-6 = -c_1 + c_2$$

$$-6 = -4 + c_2$$

$$c_2 = -6 + 4$$

$$c_2 = -2$$

So, particular solution

$$y = 4e^{-x} - 2xe^{-x} \qquad \text{OR}$$

$$y = (4 - 2x)e^{-x}$$

Answer

—— x —— x —— x ——

Question - 2 (B)

$$x^2y'' + 3xy' + y = 0$$

Solution:-

$$a = 3 \quad , \quad b = 1$$

$$m^2 + (a-1)m + b = 0$$

$$m^2 + (3-1)m + 1 = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$(m+1)(m+1) = 0$$

$$m = -1 \quad , \quad m = -1$$

Roots are equal and real.

So,

$$y = (c_1 + c_2 \ln x)\, x^m$$

Answer.

—— x —— x ——

Question - 3

$y'' + y' - 6y = 6x^3 - 3x^2 + 12x$

Solution :-

for homojenous Equation.

$a = 1 \quad , \quad b = -6$

$\lambda^2 + \lambda - 6 = 0$

$\lambda^2 + 3\lambda - 2\lambda - 6$

$\lambda(\lambda + 3) - 2(\lambda + 3)$

$(\lambda - 2)(\lambda + 3)$

$\lambda - 2 = 0, \quad \lambda + 3 = 0$

$\lambda = 2 \quad \lambda = -3$

So, $y = C_1 e^{2x} + C_2 e^{-3x}$

for choice.

$y_p = K_3 x^3 + K_2 x^2 + K_1 x + K_0$

$y_o = 3K_3 x^2 + 2K_2 x + K_1$

$y_p = 6x_3 x + 2x_2$

Now putting these values in equation.

$$\Rightarrow \frac{6\cancel{k}_3 x + 2k_2}{y''} + \frac{3k_3 x^2 + 2k_2 x + k_1}{-6\cancel{k}_3 x^3 - 6k_2 x^2 - 6k_1 x - 6k_0} - 6\cancel{k}_3 x^3 - 3k_2 x^2 + 12k$$

$$\Rightarrow -6k_3 x^3 + (3k_3 - 6k_2)x^2 + (6k_3 + 2k_2 - 6k_1)x$$

$$+ 2k_2 + k_1 - 6k_0 = 6x^3 - 3x^2 + 12x$$

$$\Rightarrow -6k_3 = 6$$

$$\Rightarrow \boxed{k_3 = -1}$$

$$3k_3 - 6k_2 = -3$$

$$3(-1) - 6k_2 = -3$$

$$-k_2 = 0 \Rightarrow k_2 = 0$$

$$6k_3 + 2k_2 - 6k_1 = 12$$

$$6(-1) + 2(0) - 6k_1 = 12$$

$$-6 + 0 - 6k_1 = 12$$

$$-6k_1 = 12 + 6$$

$$-6k_1 = 18$$

$K_1 = -\frac{18}{6}$

$\Rightarrow \boxed{K_1 = -3}$

So,

$$y_1 = -x^3 + 0x^2 - 3x - \frac{1}{2}$$

$$= -x^3 - 3x - \frac{1}{2}$$

Answer.

——— x ——— x ———

Question - 4

$$y'' - 4y' + 4y = x^2 e^{2x}$$

Solution:-

For H.E.

$$y'' - 4y' + 4y = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

$$(\lambda - 2)^2 = 0$$

$$(\lambda - 2)(\lambda - 2) = 0$$

$$\lambda_1 = 2 \quad , \quad \lambda_2 = 2$$

Roots are equal & perfect.

So,

$$y = (c_1 + c_2 x) e^{2x}$$

$$\{ y_h = c_1 e^{2x} + c_2 x e^{2x} \}$$

Now,

$$y_1 = e^{2x} \quad , \quad y_2 = x e^{2x}$$

$$y_1 = 2e^{2x} \quad , \quad y_2 = 2x e^{2x} + e^{2x}$$

$$w = y_1 y_2' - y_1' y_2$$

$$w = (e^{2x})(2xe^{2x} + e^{2x}) - (2e^{2x})(xe^{2x})$$

$$w = 2xe^{4x} + e^{4x} - 2xe^{4x}.$$

$$\{ w = e^{4x} \}$$

$$y_p = -e^{2x}$$

$$y_p = -y_1 \int \frac{y_2\, r(x)}{w}\, dx + y_2 \int \frac{y_1\, r(x)}{w}\, dx$$

$$y_p = -e^{2x} \int \frac{(xe)^{2x}(x^2e^{2x})}{e^{4x}}\, dx + xe^{2x} \int \frac{(e)^{2x}(x^2e^{2x})}{4x}\, dx.$$

$$y_p = -e^{2x} \int \frac{x^3e^{4x}}{e^{4x}}\, dx + xe^{2x} \int \frac{x^2e^{4x}}{e^{4x}}\, dx.$$

$$y_p = -e^{2x} \int x^3\, dx + xe^{2x} \int x^2\, dx.$$

$$y_p = -e^{2x}\ x^4/4 + xe^{2x}.\ x^3/3$$

$$y_p = \frac{-x^4e^{2x}}{4} + \frac{x4e^{2x}}{3}$$

$$y_p = \frac{-3x^4e^{2x} + 4x^4e^{2x}}{12}$$

$$\left\{ y_p = \frac{x^4e^{2x}}{12} \right\}$$

So,

$$y = y_h + y_p \quad \text{Answer}$$

Question - 5

$$y'' + ay' + by = 0$$

$$\Rightarrow \quad e^{-3x}$$

Solution:-

$$y_1 = e^{0x} \quad , \quad y_2 = e^{3x}$$

$$y = c_1 e^{0x} + c_2 e^{3x}$$

So, roots are equal & distinct.

$$y = (c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x})$$

So,

$$\lambda_1 = 0 \quad , \quad \lambda_2 = 3$$

$$\lambda_1 = 0 \quad , \quad \lambda_2 - 3 = 0$$

$$(\lambda)(\lambda - 3) = 0$$

So,

$$\lambda^2 - a\lambda + b = 0$$

As, $\quad a = -3 \quad , \quad b = 0$

So, $\quad y'' + ay' + by = 0$

$$y'' - 3y' = 0$$

Answer.

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