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I.D #

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Section:

"A"

Semester:

6<sup>th</sup>

Subject:

Hydraulic Engineering

Final Team

Q No: 1

Ans: 1

Solve:

The pressure drop  $\Delta p$  is expected to upon the gate opening "h" the overall depth "d" the velocity "v" density " $\rho$ " and viscosity " $\mu$ "

List of relevant variable with dimension

$$\textcircled{1} \quad \Delta p = M L^{-1} T^{-2}$$

$$\textcircled{2} \quad h = L$$

$$\textcircled{3} \quad d = L$$

$$\textcircled{4} \quad v = L T^{-1}$$

$$\textcircled{5} \quad \rho = M L^{-3}$$

$$\textcircled{6} \quad \mu = M L^{-1} T^{-1}$$

Number of variables =  $n = 6$

Number of independent dimension =  $m = 3$  (MLT)

Number of non-dimension group  $n - m = 3$

$m = 3$  scaling variables

Form dimensionless group by nondimensionalising  
the remaining variable:  $\Delta p$ ,  $h$ ,  $\mu$

$$\bar{\pi}_1 = (\Delta p)^a (U)^b (\rho)^c$$

~~$$\bar{\pi}_1 = (\Delta p)^a (U)^b (\rho)^c$$~~

$$\begin{aligned} M^0 L^0 T^0 &= (MLT^{-2})^a (LT^{-1})^b (ML^{-3})^c \\ &= M^{1+c} L^{-1+ab-3c} T^{-2-b} \end{aligned}$$

Now

$$0 = 1 + c \Rightarrow c = -1$$

$$0 = -2 - b \Rightarrow b = -2$$

$$= -1 + a + b - 3c \Rightarrow a = 1 + 3c - b$$

$$\pi_1 = \Delta p U^{-2} \rho^{-1}$$

$$\pi_1 = \frac{\Delta p}{\rho U^2}$$

$$\pi_2 = h/d \quad \text{by inspection} \\ \text{his length}$$

$$\pi_3 = \mu d^a U^b \rho^c$$

$$M^0 L^0 T^0 = (ML^{-1}T^{-1})^a (L)^b (ML^{-3})^c$$

$$= M^{1+c} L^{-1+a+b-3c} T^{-1-b}$$

$$M = 0 = -1 + c$$

$$T: 0 = 1 - b + 0$$

$$L: 0 = -1 + a + b - 3c$$

$$c = 1$$

$$b = 1$$

$$a = 1 + 3c - b$$

$$\Rightarrow \text{Now}$$

$$\Pi_3 = \mu d^4 \nu^{-1} \rho^{-1}$$

$$= \mu / \rho \nu d$$

Recognition of the Reynolds number suggest that we replace  $\Pi_3$  by

$$\Pi_3 = (\Pi_3)^{-1} = \frac{\rho \nu d}{\mu}$$

$$\boxed{1 \rho \nu d}$$

Hence dimensional analysis

$$\Pi_1 = f(\Pi_2, \Pi_3)$$

i.e

$$\frac{\Delta P}{\rho U^2} = f\left(\frac{h}{d}, \frac{\rho U d}{\mu}\right)$$

$\Rightarrow$  Dynamic similarity requires that all non-dimensional group be the same in model and prototype

i.e

$$\Pi_1 = \left(\frac{\Delta P}{\rho U^2}\right)_p$$

$$\Pi_1 = \left(\frac{\Delta P}{\rho U^2}\right)_m$$

$$\Pi_2 = \left(\frac{h}{d}\right)_p = \left(\frac{h}{d}\right)_m$$

automatic if similar shape

$$\Pi_3 = \left( \frac{\rho V d}{\mu} \right)_p = \left( \frac{\rho V d}{\mu} \right)_m$$

We have velocity ratio

$$\frac{U_p}{U_m} = \frac{(\mu \rho)_p}{(\mu \rho)_m} \times \frac{d_m}{d_p}$$

$$= \frac{0.002 / 800}{1.0 \times 10^{-6}} \times \frac{1}{5}$$

$$\boxed{\frac{U_p}{U_m} = 0.5}$$

Hence

$$U_m = \frac{U_p}{0.5}$$

$$U_m = \frac{3.0}{0.5}$$

$$\boxed{U_m = 6.0 \text{ ms}^{-1}}$$

b  $\Rightarrow$  The ratio of quantities  
of flow is

$$\frac{Q_p}{Q_m} = \frac{(\text{Velocity} \times \text{area})_p}{(\text{Velocity} \times \text{area})_m}$$

$$= \frac{V_p}{V_m} \left( \frac{d_p}{d_m} \right)^2$$

$$= 0.5 \times 5^2$$

$$\boxed{\frac{Q_p}{Q_m} = 12.5}$$

c  $\Rightarrow$  Finally, for the pressure drop

$$\pi_1 = \left( \frac{\Delta P}{\rho U^2} \right)_p = \left( \frac{\Delta P}{\rho U^2} \right)_m$$

$$= \frac{(\Delta P)_p}{(\Delta P)_m} = \frac{\rho_p}{\rho_m} \left( \frac{V_p}{V_m} \right)^2$$

$$= \frac{800}{1000} \times 0.5^2$$

$$= \frac{800}{1000} \times 0.5^2$$

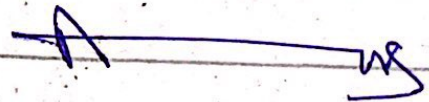
$$\boxed{\eta_1 = 0.2}$$

Hence

$$\Delta P_p = 0.2 \times \Delta P_m$$

$$\Delta P_p = 0.2 \times 60$$

$$\boxed{\Delta P_p = 12.0 \text{ kPa}}$$





Q No : 2

Given Data

Max depth of water reservoir = 78m  
 Specific Gravity of Dam =  $G = 2.25$

Allowable compression stress

For the Dam masonry =  $G_{av} = 7877 \frac{T}{m^2}$

Height of water = 1.25

$U = 0.7$

No uplift pressure  $U = 0$

Solve:

$$I: H_{\text{limiting}} = \frac{G_{av}}{\gamma_w (G - U + 1)}$$

$$= \frac{7877 \times 1000}{1000 (2.25 - 0 + 1)}$$

$$H_{\text{limiting}} = 242.153 > H_w$$

$$= 242.153 > 78$$

So it is low gravity Dam

2) Top width "a"

$$\begin{aligned} \text{Free board} &= 1.5 h_{\text{wave}} \\ &= 1.5 \times 1.25 \\ \text{F.B} &= 1.875 \text{ m} \end{aligned}$$

$$\text{Height of Dam} = HD = HW + FB$$

$$HD = 78 + 1.875$$

$$HD = 79.875$$

$$a = 14\% \text{ of } HD$$

$$a = 0.14 \times 79.875$$

$$a = 11.18$$

3) Base width = b (with out offset)

1) For no sliding criteria

$$b = \frac{HW}{UG} = \frac{78}{0.7 \times 2.25}$$

$$b = 49.5 \text{ m}$$

ii)

For no tension criteria

$$b' = \frac{Hw}{\sqrt{G}}$$

$$b' = \frac{78}{\sqrt{2.25}} = 52 \text{ m}$$

$$\text{Use } b = 52 \text{ m}$$

4) Depth of vertical portion  
on U/s side

$$h' = 2a \sqrt{G - c_u}$$
$$= 2(11.18) \sqrt{2.25 - 0}$$

$$h' = 33.54 \text{ m}$$

P.T.O.

5) Up stream offset

$$\Rightarrow \frac{a}{16}$$

$$= \frac{11.18}{16}$$

$$= 0.699$$

$$= 0.7 \text{ m}$$

5) Depth below the water level to the end of inclined portion is  $U/s = 3.14 a \sqrt{G}$

$$U/s = 3.14 a \sqrt{G}$$

$$= 3.14 (11.18 \sqrt{2.25})$$

$$= 3.14 (11.18 \sqrt{2.25})$$

$$= 52.65$$

$$U/s \approx 52$$

7 Total width

$$b = b' + \frac{a}{10}$$

$$= 52 + \frac{11.18}{16}$$

$$b = 52.69 \text{ m}$$

$$b \approx 52.7 \text{ m}$$

$$8) \tan \theta = \frac{b'}{H} = \frac{52}{78}$$

$$\tan \theta = \left( \frac{2}{3} \right)$$

$$\theta = \tan^{-1} \left( \frac{2}{3} \right)$$

$$\theta = 33.69^\circ$$

$$\boxed{\theta = 33.69^\circ}$$

So

9) Depth of vertical portion on  $\frac{D}{s}$   
From  $W_c$  on  $\frac{U}{s}$  side

$$\tan \theta = \frac{d}{d'}$$

$$\tan \theta = \frac{11.18}{d'}$$

$$\tan \theta = \frac{11.18}{d'} \Rightarrow \tan \theta d' = 11.18$$

$$\frac{2}{3} d' = 11.18$$

$$d' = 11.18 \times \frac{3}{2}$$

$$d' = \cancel{11.18}$$

$$d' = 16.77 \text{ m}$$

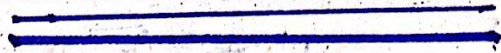
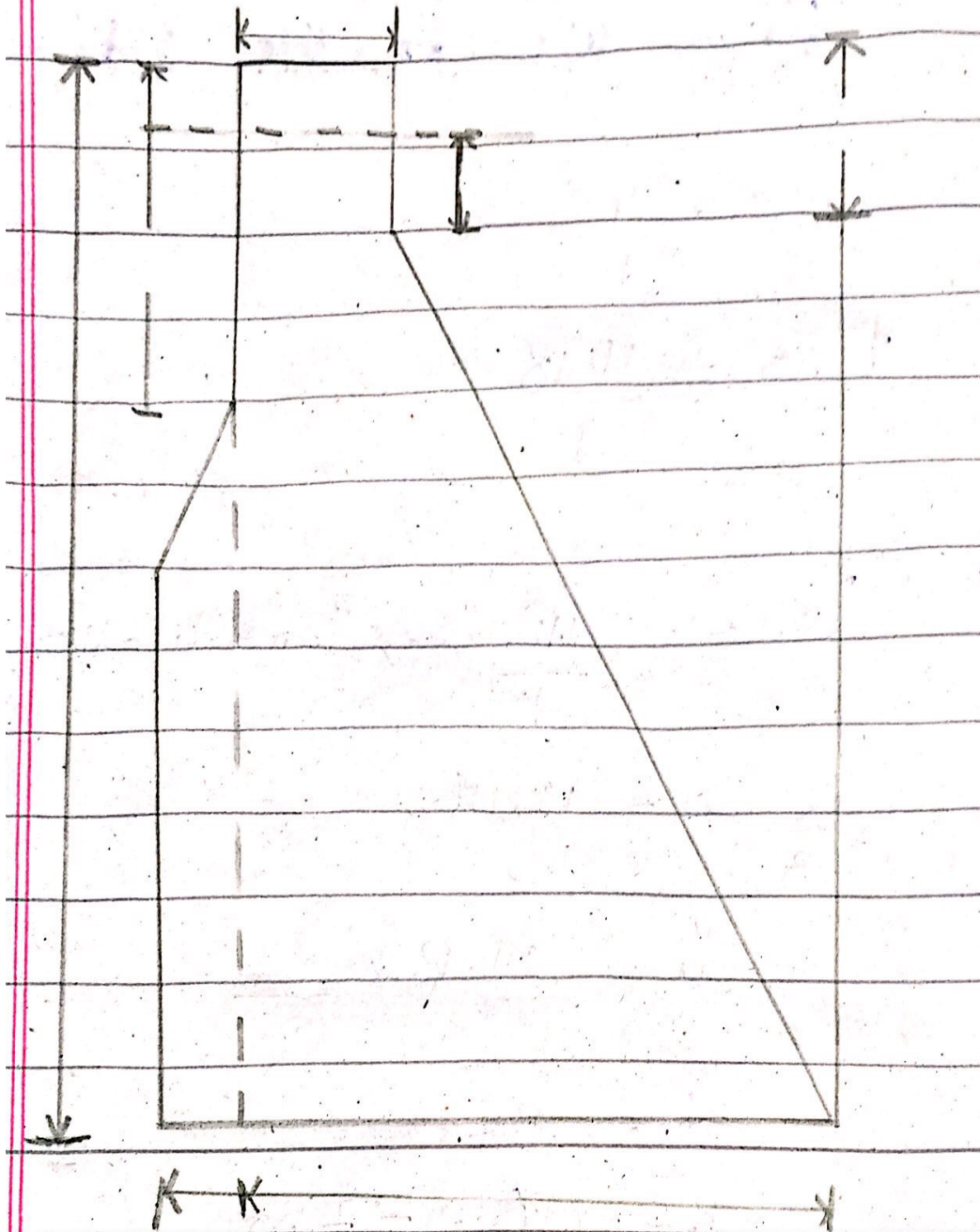
Depth on vertical portion

$$d = d' + FB'$$

$$d = 16.77 + 1.875$$

$$d = 18.645$$

$$\boxed{d = 18.64} \quad A \quad \underline{\underline{g}}$$



Q No: 4

## Fall velocity:

When a grain fall down in still water it obtain a constant velocity when the upward fluid drag force on the grain is equal to the downward submerged weight of the grain.

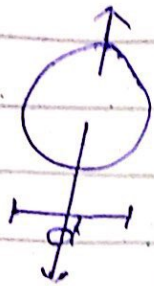
This constant velocity is defined as the fall velocity of the grain.

This is also called settling velocity.

## Fall velocity depends on

- ① Particle diameter
- ② Particle density
- ③ Particle concentration
- ④ Particle shape
- ⑤ Viscosity of water
- ⑥ Turbulancey





$w_s$  = Fall velocity

submerged weight

The force balance b/w the drag force and the submerged weight given

$F_D$  = submerged weight

$$\frac{1}{2} \rho C_D \frac{\pi d^2}{4} w_s^2 = (\rho_s - \rho) g \frac{\pi d^3}{6}$$

$$A = \frac{\pi d^2}{4} \Rightarrow \text{Project Area}$$

$C_D$  = Drag Coefficient

$w_s$  = fall velocity of sediment

$$w_s = \sqrt{\frac{4gd}{3C_D} \left( \frac{\rho_s - \rho}{\rho} \right)}$$

$\rho$  = density of water

$\rho_s$  = density of sediment particle

## 1. Particle Diameter:

The diameter of a sphere particle uniform settling velocity as the given particle in the same sedimentation.

## 2. Particle density

Particle density affected the settling fall velocity.

As air density increases with decreasing altitude at about 1% per 80 meter (260 ft) for every 160 meter of fall the terminal speed decrease 1%.

## 3. Particle shape:

Non-spherical analoges particles fall up to 75% slower than equivalent sphere model.

Show 100  $\mu\text{m}$  non spherical particles to travel 44% further than sphere vertical structure of modelled velocity volcanic ash cloud is sensitive to particle shape.

#### 4). Viscosity of Water:

Fluid velocity through porous media is approximated as inversely proportional to the kinematic viscosity


A decrease in viscosity therefore increase the velocity of a compound through porous media.

#### 5). Turbulance of Water:

Turbulancy of water effect the fall velocity of water reservoir because the non-linearity and zig zag path effect

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the flow of water to cause  
the variation in the flow.



Q/NO: 3

## Dimensional Analysis

Where all of the variables ( $x$ ) are dimensional  
 $f(x_1, x_2, \dots, x_n) = 0$

Then the above phenomenon can be represented as

$$W(\pi_1, \pi_2, \dots, \pi_m) = 0$$

~~Where~~ where all variable ( $\pi$ ) are non-dimensional

The nature of  $f$  and  $W$  may be obtained from experiment

## Buckingham Pi Theorem

$$f(x_1, x_2, \dots, x_n) = 0 \rightarrow W(\pi_1, \pi_2, \dots, \pi_m) = 0$$

Where  $m < n$ ;  $m = n - k$

## Experiment show, For viscous flow

$$f = (V, g, h, \nu) = 0$$

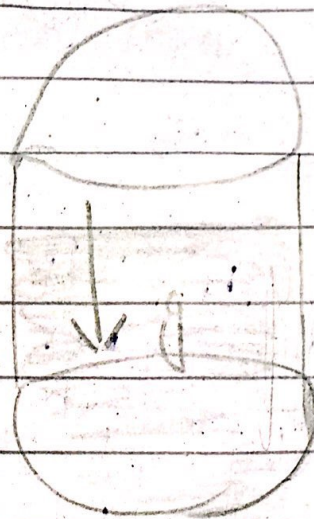
	M	L	T
V	0	1	-1
g	0	1	-2
h	0	1	0
$\nu$	0	2	-1

$$m = 3$$

$$k = 2$$

$$n = 4$$

Let's take the  
repeating variables  
g, h



Non repeating variable

$$V, \nu$$

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	<u>L</u>	<u>T</u>
U	1	-1
g	1	-2
h	1	0
v	2	-1

$$\mathcal{F}(U, g, h, v) = 0$$

$$n = 4 \quad k = 2 \quad m = 2$$

$$\text{Repl. var} = g, h$$

$$\text{Non-repl} = U, v$$

$$\overline{\Pi}_1 = x_{n1} (x_{r1})^{a_{11}} (x_{r2})^{a_{12}} (x_{r3})^{a_{13}} \dots (x_{rk})^{a_{1k}}$$

$$\overline{\Pi}_1 = U(g^a h^b)^5$$

$$L^0 = L^1 (L^{-2})^a (L)^b = L^{1+a+b} = L^{-2a}$$

$$= a = b = -1/2$$

$$\overline{\Pi}_1 = \frac{U}{\sqrt{gh}}$$

Similarly

$$\pi_2 = U(g^a)(h^b)$$

$$L^0 T^0 = L^{2a-1} (L T^{-2})^a (L)^b$$

$$2 + a + b = 0 \quad -1 = -2a$$

$$a = -1/2, \quad b = -3/2$$

$$\pi_2 = U/\sqrt{gh^3}$$

$$\frac{U}{\sqrt{gh}} = Fr \quad \text{Froude number} \quad \frac{U}{\sqrt{gh}} = \frac{U}{\sqrt{gh}} \frac{U}{U} = \frac{Fr}{Re}$$

We Also know

$$f_2(Fr, Fr/Re) = 0$$

$$Fr = W(Fr/Re)$$

$$Fr = \text{constant}$$



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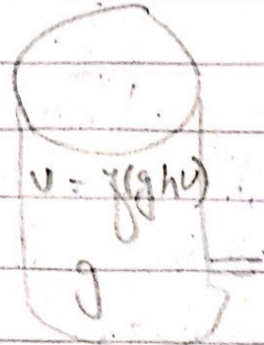
$$F_r = V \cdot (F_r / R_e)$$

Experiment necessary to find the nature of junction

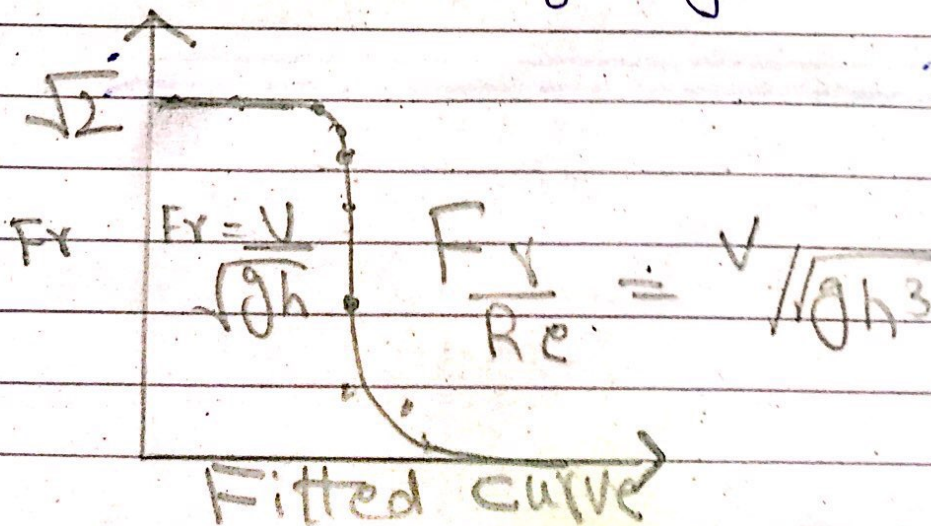
## Dimensional Analysis

I would like to do a laboratory experiment

We can vary  $h$  and measure  $V$  for few cases



Finally we do a curve fitting



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## Conclusion

⇒ One figure is enough as opposed to many fig of dimensional system

⇒ Tank size is irrelevant as long as our assumption aren't violated

The plot developed from experiment in a small model may be used for similar bigger tank