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7720

SECTION B

INTRO TO STRUCTURAL DYNAMICS & EARTHQUAKE ENGG

FINAL ASSIGNMENT

Question No 1:-

The general E.O.M for SDOF system is

$$Ku + c\dot{u} + m\ddot{u} = P(t)$$

In our case system is undamped ($c=0$) undergoing Free vibration ($P(t)=0$).

Hence general EOM become $ku + m\ddot{u} = 0$ ---(1)

$$K = 3EI/L^3$$

$$= \frac{3 \times 29000 \times 150}{(10 \times 12)^3}$$

$$= 7.55 \text{ k/in} \Rightarrow k = 7.55208$$

In order to eliminate the chances of mistake during calculation, it is more appropriate to use Fundamental units like lb, ft sec or kg m, sec.

$$K = 7.55 \text{ K/in} = 90625 \text{ lb/ft}$$

$$m = 7720/32.2 = 240.9 \text{ slug}$$

$\omega_n = \sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{90625}{240.9}}$

$W_n = 19.42$ rad/sec

$T_n = \frac{2\pi}{\omega_n} \Rightarrow \frac{2\pi}{19.42} = 0.323$ sec

put m & k in eqⁿ (i)

$90625u + 240.9\ddot{u} = 0$

where k is in lb/ft & m is in slug $lb\,sec/ft^2$

⇒ General solution to EOM for a undamped free vibration is

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{2\omega_n}\sin(\omega_n t)$$

$U(0) = \frac{1}{2''} = \frac{1}{24}$ ft & $\dot{u}(0) = 0$

$U(t) = \frac{1}{24} \times \cos(19.425\,t) + 0 = \left(\frac{1}{24}\right) \times \cos(19.42)t$

Equivalent static force at any time 't' is

$$F_s(t) = k \cdot u(t) = \frac{90625 \times \cos(19.42t)}{24}$$

$$= 3776 \cos(19.42t)$$

Amplitude of dynamic displacement, U_0 for undamped free vibration is

$$U_0 = \sqrt{[(u(0))^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2]}$$

$$= \sqrt{\left(\left(\frac{1}{24}\right)^2 + 0\right.}$$

$$= \frac{1}{24} ft$$

Amplitude of equivalent static force, F_{so}

$$K_{\theta_0} = 90625 \times \frac{1}{24}$$

$$= 3776.$$

Question No 2

ξ (Damping ratio) of reinforced concrete with considerable cracking = 3-5%
= 3%

Using Data of beam given in Question # 1

Required:-

Develop & solve the equation showing variation in Equivalent static force with time. Draw graph to show the variation of displacement with time & the variation of equivalent static force with time.

Sol:-

E.O.M for damped free variation is

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \text{--- (1)}$$

from question 1

$K = 90625$ lb/ft & $m = 240.9$ lb sec²/ft

$w_n = 19.42$ rad/sec

$$C = \xi \times 2 m w_n$$

$$C = (0.03) \times 2(240.9 \times 19.42)$$

$$C = 280.69 \text{ lb.sec/ft}$$

Put values in eq (1)

$$90625u + 280.69 + 240.9\ddot{u} =$$

Solution to the EOM for damped free vibration is

$$u(t) = e^{-\xi \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left(\dot{U}(0) + U(0)\xi\omega_n\right)\sin \omega_D t\right]$$

$$\omega_D = 19.42 \text{ rad/sec}$$

$$U(t) = e^{-0.03 \times 19.42 t}\left[\frac{1}{24} \times \cos(19.42t) + \frac{1}{19.42} \times \left(0 + \frac{1}{24} \times 0.03 \times 19.42\right) \times \sin(19.42t)\right]$$

$$u(t) = e^{-0.582t}\left(0.041 \times \cos(19.42t) + 0.00125 \times \sin(19.42t)\right)$$

$$F_s(t) = k \cdot U(t) \Rightarrow 90625 \times u(t)$$

$$F_s(t) = e^{-0.582t}\left((90625 \times 0.041)\cos(19.42t) + (90625 \times 0.00125) \times \sin(19.42t)\right)$$

$$F_s(t) = e^{-0.582t}\left(3715.62\cos(19.42t) + 113.28\sin(19.42)t\right)$$

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Question No 3

Given Data:-

Force = 60 kips

Displacement of tank = $\frac{TD}{1000} = \left(\frac{7720}{1000}\right)'' = 7.720''$

time taken to complete 7 cycles = 3.57 sec

amplitude of displacement = 2.286cm = 0.9"

Required Data:-

1) Damping ratio.
2) Natural period of undamped vibration
3) Stiffness of structures
4) Weight of tank.
(5) Damping co-efficient
(6) No. of cycles to reduce the displacement amplitude to 0.5"

Sol:-

→ Displacement of tank, $u_1 = 7.720''$

→ After 7 cycles i-e after $j=7$, $u_{j+1} = u_8 = 0.9''$

a) Damping ratio = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$7 = \frac{1}{2\pi\zeta} \ln\left[\frac{7.720}{0.9}\right]$$

$$\zeta = 0.048 = 4.8\%$$

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b) Natural period of undamped vibration: $T_n = ?$

As, The 7 cycles of vibrations are completed in 3.57 sec

=) Time required to complete on cycle, T_D

$$= \frac{3.57}{7} = 0.51 \text{ sec}$$

Now,

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{(\omega_n\sqrt{1-\zeta^2})}$$

$$= T_D = \frac{T_n}{(1-\zeta)^2}$$

$$= T_n = T_D \times \sqrt{1-\zeta^2}$$

$$= 0.51 \times \sqrt{1-(0.0488)^2}$$

$$= 0.5094$$

$$= 0.51 \text{ sec}$$

$$T_n = 0.51 \text{ sec}$$

c) Stiffness of Structure, $k = ?$

$$k = \frac{60 \times \omega_s 60}{7.758} = 3.91 \text{ k/in}$$

$$k = 3.91 \text{ k/in} = 46920 \text{ lb/ft}$$

d) Weight of Tank, $w = ?$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{\frac{w}{g}}} = \sqrt{\frac{k \cdot g}{w}}$$

$$\Rightarrow \omega_n^2 = \frac{k \times g}{w}$$

$$w = \frac{k \times g}{\omega_n^2}$$

Also $\omega_n = \frac{2\pi}{T_n}$

$$w = \frac{kg}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$= kg \times \frac{T_n^2}{4\pi^2}$$

$$w = \left[\frac{46920 \text{ lb}}{ft} \times \frac{32.2 \text{ ft}}{sec^2}\right] \times \frac{(0.51 \text{ sec})^2}{4\pi^2}$$

$$= 9953.93 \text{ lb} = 9.95 \text{ k}$$

e) Damping Co-efficient, $C = ?$

It is known that,

$$\zeta = \frac{c}{2m\omega_n}$$

$$\Rightarrow C = \zeta \times 2m\omega_n$$

$$= \zeta \times 2m \times \frac{2\pi}{T_n}$$

$$= \frac{0.0488 \times 4 \times \pi \times (9953.93/32.2)}{0.51}$$

$C = 371.71$ lbsec/ft

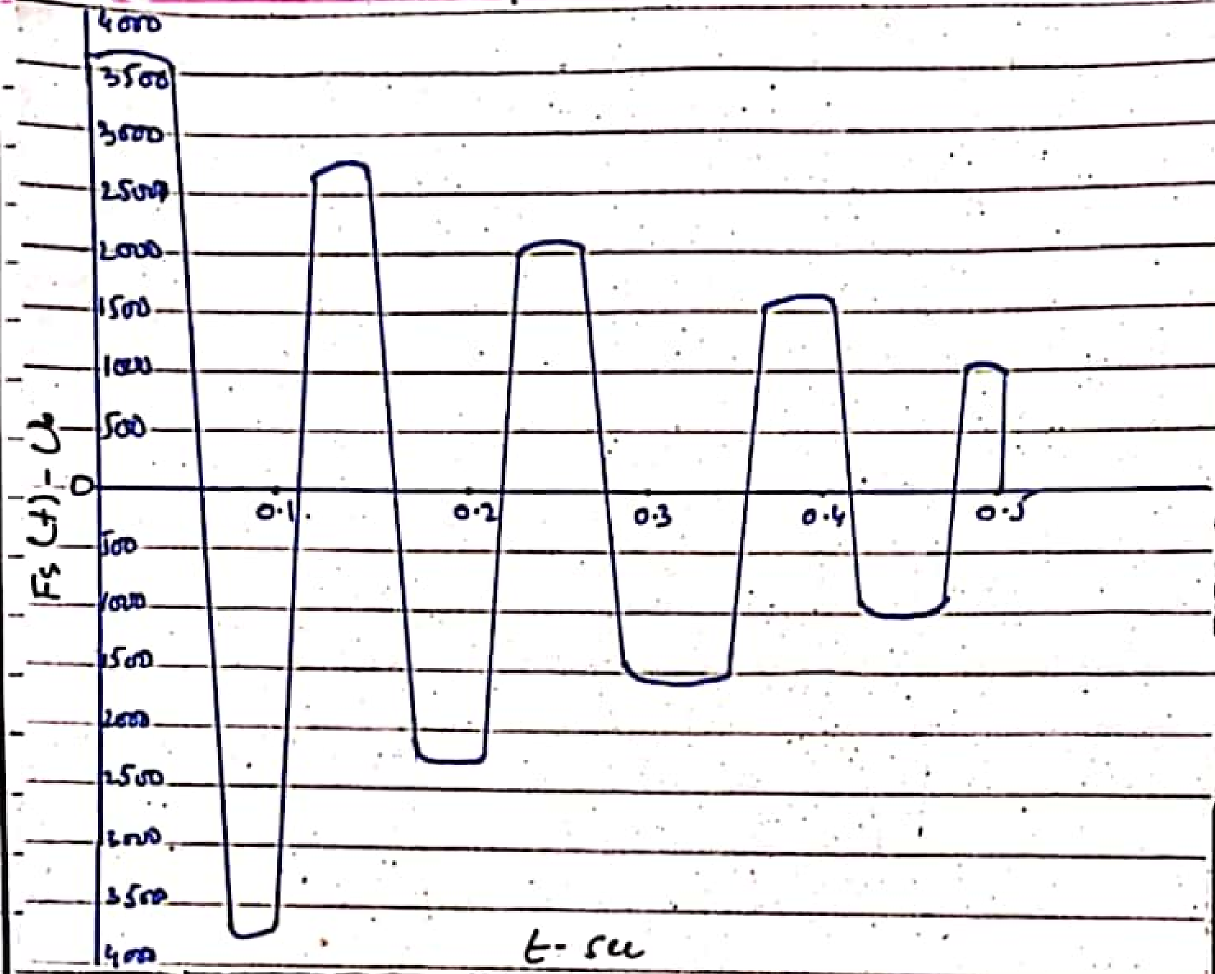
8) No. of cycles to reduce the displacement amplitude to 0.5", j = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

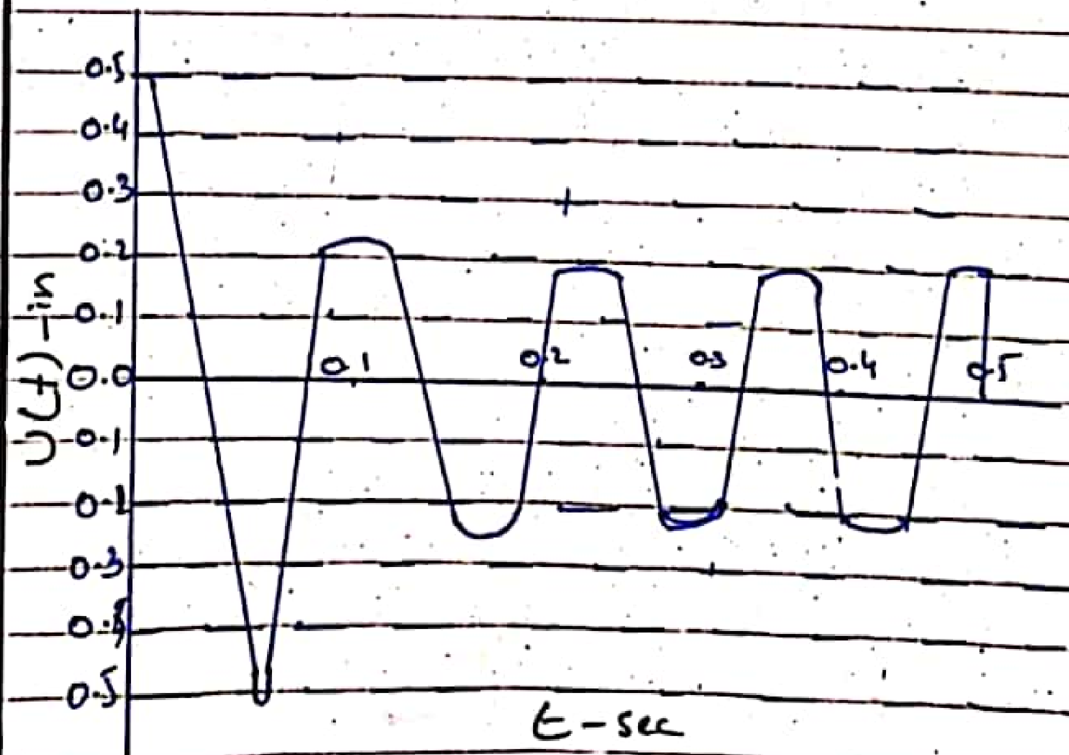
$$\Rightarrow j = \frac{1}{2\pi \times 0.0488} \ln\left[\frac{7.758}{0.5}\right]$$

$$j = 8.91 \text{ or } 9 \text{ cycles}$$

Question & Graphs. Damped Free vibration

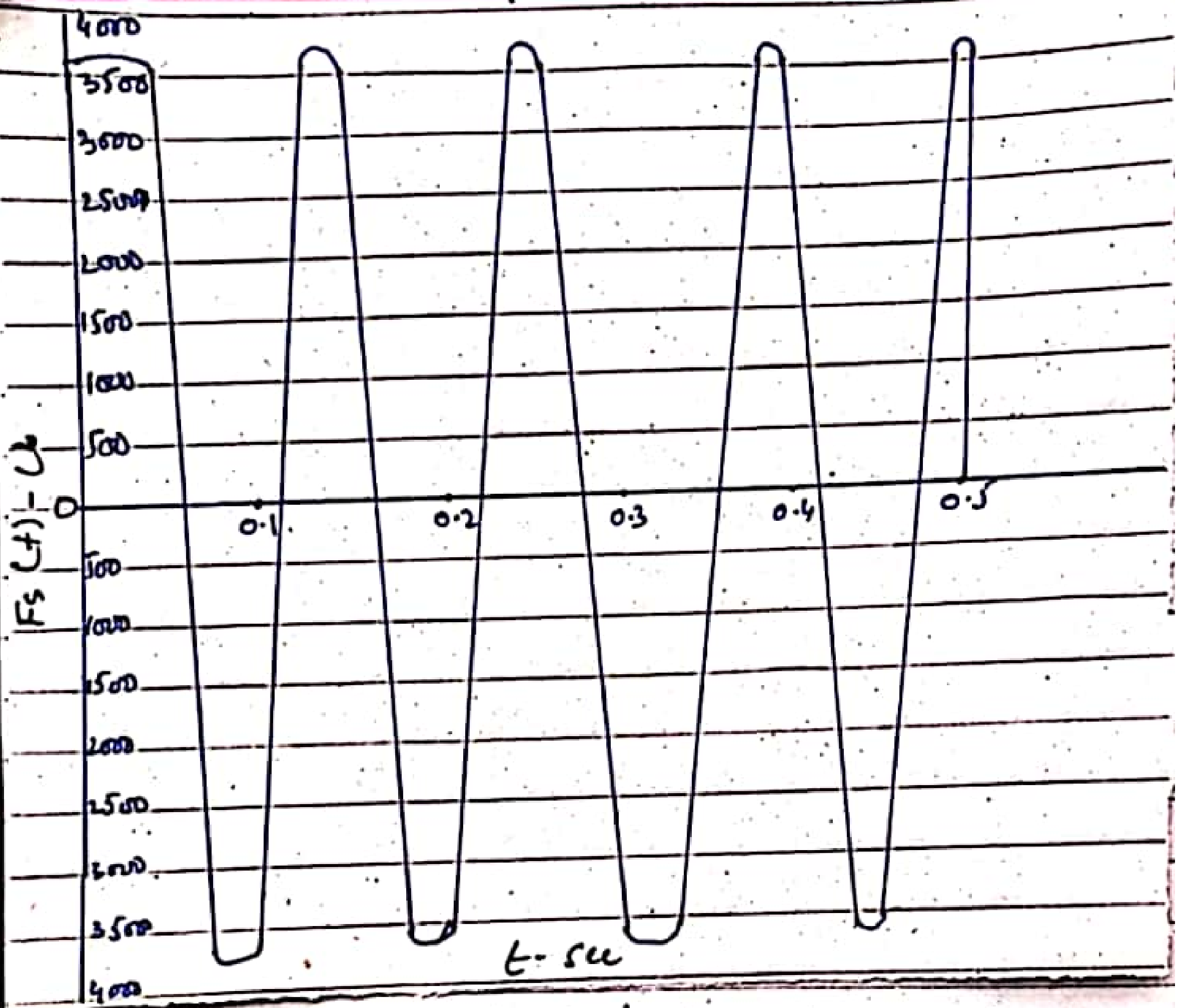


Variation of equivalent static forces with time.

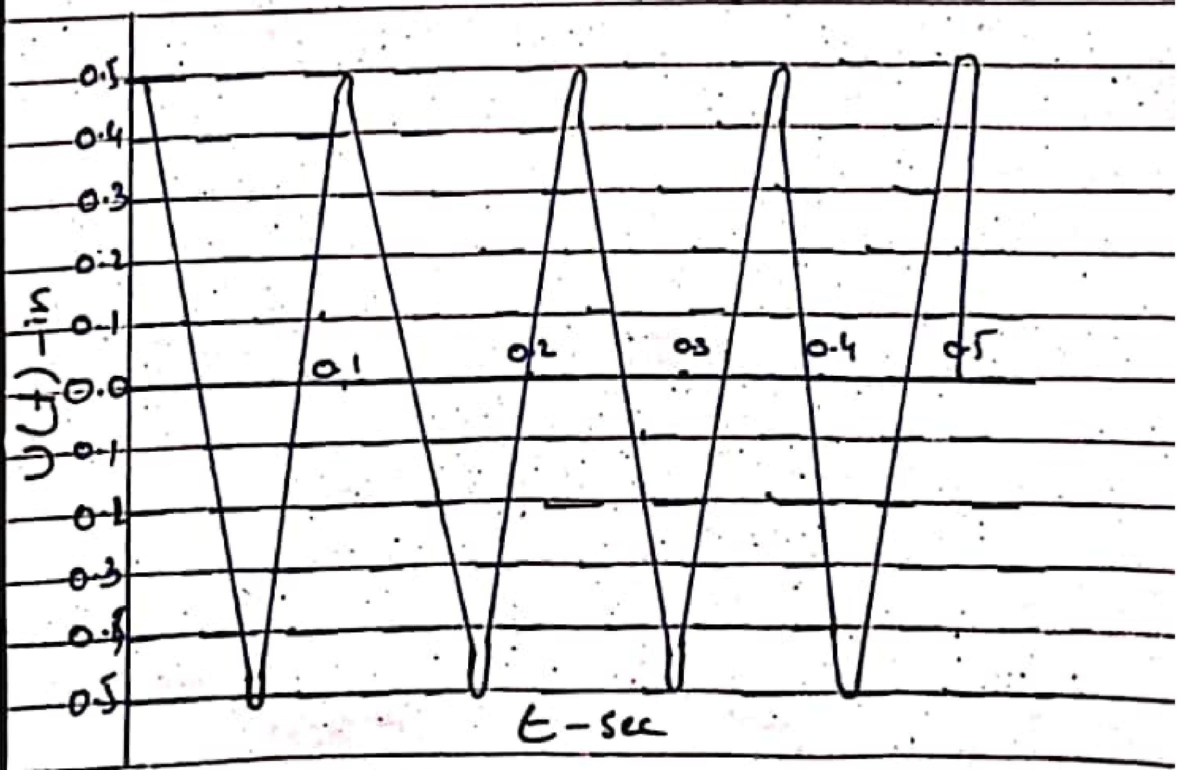


Variation of displacement with time

Question 1 Graphs.



Variation of equivalent static forces with time.



Variation of displacement with time