Question: 1a $\boxed{y' = (x+2)y^2}$

Put (x) $y' = (x+2)y^2$

Sol:

$$y' = (x+2)y^2$$

$$\frac{dy}{dx} = (x+2)y^2$$

$$\int \frac{1}{y^2}\, dy = \int (x+2)\, dx$$

$$\int y^{-2}\, dy = \int (x+2)\, dx$$

$$\frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} + 2x + C$$

$$\frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} + 2x + C$$

$$\frac{y^{-1}}{-1} = \frac{x^2}{2} + 2x + C$$

using both side by -1

$$y^{-1} = -\left(\frac{x^2}{2} + 2x + C\right)$$

$$y = -\left(\frac{1}{x^2/2 + 2x + C}\right).$$

Question 1(b):-

$$y' = (y + 9x)^2$$

Sol:- $y' = (y + ax) \longrightarrow (a)$

let suppose

$$y + 9x = v \longrightarrow (1)$$

then eq (1) will become

$$\frac{dv}{dx} - 9 = v^2$$

$$\frac{dv}{dx} = v^2 + 9$$

$$\int dx = \int \frac{1}{v^2 + 9}\, dv$$

$$\int dx = \int \frac{1}{v^2 + 3^2}\, dv$$

$$x + c_1 = \frac{1}{3}\tan^{-1}(v/3)$$

$$3x + 3c = \tan^{-1}(v/3)$$

$$3x + 3c = \tan^{-1}(v/3)$$

$$v/3 = \tan(3x + c)$$

$$v = 3\tan(3x + c)$$

$$y + 9x = 3\tan(3x + c)$$

$$y = -9x + 3\tan(x + c)$$

Q NO 2

$$M^3\,du + y^3\,dy = 0$$

$$M\,du + N\,dy = 0$$

$$M = M^3, \quad N = y^3$$

$$\Rightarrow \frac{\partial M}{\partial y} = 0 \qquad \frac{\partial N}{\partial M} = 0$$

$$u = \int M\,du + K(y)$$

$$u = \int M^3\,du + K(y)$$

$$u = \frac{M^4}{4} + K(y) \longrightarrow (1)$$

$$\frac{\partial u}{\partial y} = 0 + d/dy\,(y)$$

Since we know that

$$\frac{\partial u}{\partial y} = N = y^3$$

$$y^3 = d/dy\ K(y) \Rightarrow \int y^3 = \int dy$$

P.T.O

$\Rightarrow k(y) = y^4/4 + c_1$ put in 1

$$u = \frac{x^4}{4} + y^4/4 + 1$$

$$c_2 = \frac{x^4}{4} + y^4/4 = c_1$$

$$c_2 - c_1 = x^4/4 + y^4/4$$

$$c = x^4/4 + y^4/4 = u \text{ Ans}$$

Q3 (a)

$$4y'' - 20y + 25y = 0$$

dividing by (4)

$$\frac{4y''}{4} - \frac{20y'}{4} + \frac{25y}{4} = 0$$

$$y'' = \frac{20y'}{4} + \frac{25y}{4} = 0$$

$$y'' - 5y' + \frac{25y}{4} = 0$$

$a = -5$, $b = 25/4$

auxilary

$$m^2 + 5m + \frac{25}{4} = 0$$

Formula ~~$x^2 - 2xb + b^2 = 0$~~

$$m^2 - 5m + (5/2)^2 = 0$$

$$x - 5/2\,(5/2)^2 = 0$$

$$(x - 5/2)^2 = 0$$

$$(\lambda - 5/2)(\lambda - 5/2) = 0$$

$$\lambda_1 = 5/4 \quad , \quad \lambda_2 = 5/4$$

Same & real root

So,

$$y = (C_1 + C_2 x)e^{5/4\,x} = (C_1 + C_2 x)e^{5/2\,x}$$

Question 3b :-

$$4y'' - 6y' - 7y = 0$$

Sol :- $4y'' - 6y' - 7y = 0$

Auxilary eq: is

$$4\lambda^2 - 6\lambda - 7 = 0$$

So by Quadratic Formula

Here,

$$a = 4, \quad b = -6, \quad c = -7$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\lambda = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(-7)}}{2(4)}$$

$$\lambda = \frac{6 \pm \sqrt{36 + 112}}{8}$$

$$\lambda = \frac{6 \pm \sqrt{148}}{8}$$

taking 2 as common

$$\lambda = \frac{3 \pm \sqrt{37}}{4}$$