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Final Term Assignment Linear Algebra.

Q4. part d :- compute det A for below 3x3 matrix.

$$A = \begin{bmatrix} ID1 & ID1 & ID1 \\ ID2 & ID3 & ID2 \\ ID4 & ID1 & ID5 \end{bmatrix} \quad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

Sol:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 4 & 1 & 5 \end{bmatrix}$$

Rule for determinent 3x3
if

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$|A| = a(ei - fh) - b(di - fg) + c(dh - eg)$$

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Now $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 4 & 1 & 5 \end{bmatrix}$ $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

$|A| = 1(3\times5-2\times1) - 1(2\times5-4\times2) + 1(2\times1-4\times3)$

$|A| = 1(15-2) - 1(10-8) + 1(2-12)$

$|A| = 1(13) - 1(2) + 1(-10)$

$|A| = 13 - 2 - 10$

$|A| = 13 - 12$

$|A| = 1$ Ans:

Q4: Part (i): write down 2x2 bit Matrix with determinant 0.

Sol: 2x2 matrix is follow.

$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

$|A| = \det \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix}$

$|A| = 2\times2 - 4\times1$

$|A| = 4 - 4$

$|A| = 0$

This Answer of two by two matrix with determinent 0.

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Q4: **Part b:** write down all 2x2 bit matrices with determinant 1. (Remember bits are either 0 or 1 and 1+1=0 bits)

Sol: 2x2 matrix with determinant 1

$$A = \begin{bmatrix} 3 & 1 \\ 8 & 3 \end{bmatrix}$$

$$|A| = \det \begin{vmatrix} 3 & 1 \\ 8 & 3 \end{vmatrix}$$

$$|A| = 3 \times 3 - 8 \times 1$$
$$|A| = 9 - 8$$
$$|A| = 1$$

This is the answer of two by two matrix with determinant 1.

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Q4: Determinants: let $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2x2 matrix.

Part A: For which values of det M does have an inverse?

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Change the letters with numbers.

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$$M = \begin{bmatrix} -5 & 2 \\ 4 & 3 \end{bmatrix}$$

$$|M| = \det \begin{vmatrix} -5 & 2 \\ 4 & 3 \end{vmatrix}$$

$$|M| = -5 \times 3 - 4 \times 2$$
$$|M| = -15 - 8$$
$$|M| = -23$$

These values of det of M have inverse values.

Q3: what are the four main things we need to define for a vector space? which of the following is a vector space over R? For those that are not vector space modify one part of the determinant to make it into a vector space.

Ans: vector Space: A vector space is a set V on which two operations + and • are defined, called vector addition and scalar Multiplication.

Four main things vector addition.

(1) Commutative law. For all vectors u, and v in V, $u+v = v+u$.

(2) Associative law: For all vectors u, v, w in V, $u+(v+w) = (u+v)+w$

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(3) **Additive identity:** The set V contains an additive identity element, denoted by 0, such that for any vector v in V, $0+v=v$ and $v+0=v$.

(4) **Additive inverse:** For each vector v in V, the equations $v+x=0$ and $x+v=0$ have solution x in V, called an additive inverse of v, and denoted by $-v$.

For main things for scalar multiplication.

(1) **Distribution law:** For all real numbers c and all vectors u, v in V, $c\cdot(u+v)=c\cdot u+c\cdot v$.

(2) **Distributive law:** For all real numbers c, d, and all vectors v, in V
$(c+d)\cdot v=c\cdot v+d\cdot v$

(3) **Associative law:** For all real numbers c, d and all vectors v in V, $c\cdot(d\cdot v)=(cd)\cdot v$

(4) **Unitary law:** For all vectors v in V
$1\cdot v=v$.

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Q3: **Part b:** V = {polynomials with complex coefficients of degrees ≤ 3} with usual addition and scalar multiplication of polynomials.

Ans: The set of polynomials of degree n should be a vector space because.

① There is an "one" and a "zero" in this set.

② we can find inverse for addition and multiplication from this set.

③ It follows all axioms of addition.

④ It follows all the axioms of scalar multiplication.

Part a: V = {2x2 matrices with entries in R} usual matrix addition and

$$k \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ka & b \\ kc & d \end{pmatrix} \text{ for } k \in R.$$

Sol:- The set V of all matrices of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $a, b \in R$, over R with addition ⊕ and scalar multiplication ⊙ defined as.

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$$k \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & b \\ kc & d \end{bmatrix} \text{ for } k \in R$$

we claim that R is indeed a vector space with the given operations. note first that R, is closed under the addition and scalar multiplication operations: for.

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Q1 consider the following vectors R^3.

$$v_1 = \begin{bmatrix} ID1 \\ ID2 \\ ID3 \end{bmatrix}, v_2 = \begin{bmatrix} ID2 \\ ID3 \\ ID4 \end{bmatrix}, v_3 = \begin{bmatrix} ID3 \\ ID4 \\ ID5 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, v_3 = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

Sol:

consider the set of vectors.

$$\left\{ v_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, v_2 \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, v_3 \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \right\}$$

Since R^3 need 3 linearly independent vectors.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

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(8)

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$ Now apply row operation.

$R_1 - R_2$.

$$\mathcal{R} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 4 & 5 \end{bmatrix} \quad 3R_2 - R_3 .$$

$$\mathcal{R} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix} \quad R_3 - 3R_1$$

$$\mathcal{R} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix} \quad \text{now } R_3 - R_1$$

$$\mathcal{R} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} \quad R_1 + 3R_3$$

$$\mathcal{R} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} \quad R_1 - 2R_2$$

$$\mathcal{R} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} \quad R_2 + 2R_3$$

$$\mathcal{R} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow 3 \text{ pivots}$$

It shows that these $v_1 \begin{bmatrix} ID1 \\ ID2 \\ ID3 \end{bmatrix}$, $v_2 \begin{bmatrix} ID2 \\ ID3 \\ ID4 \end{bmatrix}$, $v_3 \begin{bmatrix} ID3 \\ ID4 \\ ID5 \end{bmatrix}$.

are linearly independent vectors.