SYED JAWWAD
7386

EARTHQUAKE

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Q #1

Given Data:-

$E = 29000\,ksi$, $I_c = 150$

δ = Deflection Due to 7386 lb static load

Beam is pulled 1/2" downwards

REQUIRED:-

Natural time Period of System developed & Equation of Motion

Draw Graphs to show the Variation of Displacement with time & the Variation of Equivalent Static forces with time:.

SOLUTION:-

The General E.O.M of SDOF System is

$$ku + c\dot{u} + m\ddot{u} = P(t)$$

In our Case Undamped ($c=0$) undergoing free vibration ($P(t)=0$)

Hence General E.O.M become $ku + m\ddot{u} = 0$ ...... (i)

$$k = 3EI/L^3$$

$$= \frac{3 \times 29000\,k/in^2 \times 150\,in^4}{(10 \times 12\,in)^3}$$

$$k = 7.55\,k/in$$

In order to Eliminate the Chance of Mistake during Calculation, It is more appropriate to Use fundamental units like lb, ft, sec or kg, m, sec

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$k = 7.55\ k/in = 90625\ lb/ft$

$w_n = \sqrt{k/m} \Rightarrow \sqrt{\frac{90625}{m}}$

$m = \frac{7386\ lb/sec^2}{32.2\ ft} = 229.37\ slug$

Now put the value of m in wn formula

$w_n = \sqrt{\frac{90625}{229.37}} = 19.87\ rad/sec$

$T_n = \frac{2\pi}{w_n} = \frac{2\pi}{19.87} = 0.316\ sec$

Substituting the corresponding values in Equation 1

$90625u + 229.37\ddot{u} = 0$

where k in lb/ft and "m" is in lb sec/ft^2

General solution to the E.O.M for undamped free vibration is

$u(t) = u(0)\cos(w_n t) + \dot{u}(0)/w_n \sin(w_n t)$

$u(0) = \frac{1}{2''} = \frac{1}{24}$ ft and $\dot{u}(0) = 0$

$u(t) = \left(1/24\right)\cos(19.45t) + 0 = (1/24)\cos(19.45t)$

Equivalent static force at anytime "t" is

$f_s(t) = k \cdot u(t) = \frac{90625\cos(19.45t)}{24}$

$f_s(t) = 3776\cos(19.45t)$

Amplitude of Dynamic Displacement, u_o for undamped free vibration is

$$u_0 = \sqrt{\left[[u(0)]^2 + \left(\frac{\dot{u}(0)}{\omega_n} \right)^2 \right]}$$

$$\sqrt{\left(1/24\right)^2 + 0} = 1/24 \text{ ft}$$

Amplitude for Equivalent Static force

$$k u^0 = 90625 \times \frac{1}{24} = 3776 \text{ lb}$$

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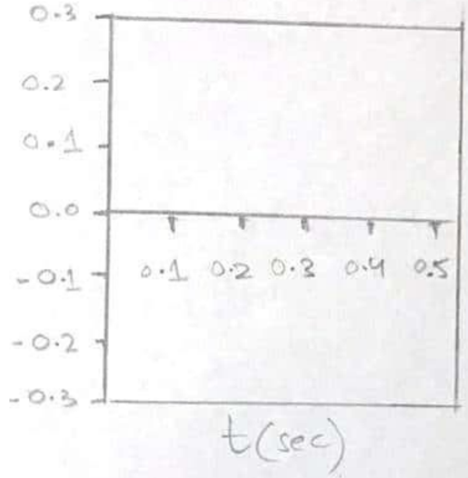
Graph Representing Variation of Displacement with Time

(4)



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Q#2

Given Data:-

S (Damping Ratio) of Reinforced Concrete with Considerable Crack = 3-5%
= 3%

Using Data of beam given question 1.

Required:-

→ Develop and Solve the Equation showing variation in Equivalent Static force with time

→ Draw graph to show the variation of Displacement with time and the variation of Equivalent Static force with time.

Solution:-

E.O.M for damped free vibrations is

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \text{— (1)}$$

from Question 1

$k = 90625$ lb/ft

$m = 229.37$ lb sec²

$w_n = 19.87$ rad/sec

$C = \xi \times 2 m w_n$

$C = (0.03) \times 2 (229.37)(19.87)$

$C = 273.45$ lb. Sec/ft

Put values in Eq (1)

$$90625u + 273.45 + 229.37u = 0$$

Solution ~~for~~ of EOM for Damped free Vibration is.

$u(t) = e^{-\xi \omega_n t}\left(u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0)\xi\,\omega_n\right]\sin\omega_D t\right)$

$\omega_D = 19.87$ rad Sec

$u(t) = e^{-(0.03\times 19.87t)}\left[\frac{1}{24}\times \cos (19.87t) + \frac{1}{19.87}\left(0 + \frac{1}{24}\times 0.03 \times 19.87 \times \sin(19.87)\right)\right.$

$u(t) = e^{-0.596t}\left[0.041\cos(19.87) + 0.050\,(0.024\sin(19.87))\right)$

$= e^{-0.596t}\left[\left[0.041\cos(19.87 + 0.0012\times \sin(19.87t)\right)\right]$

~~$= e^{-0.596t}(0.021)*$~~

$f_s(t) = ku(t) = 90625 \times u(t)$

$f_s(t) = e^{-0.596t}\left[(90625\times 0.041)\cos(19.87t) + (90625\times 0.0012)\sin(19.87t)\right)$

$= e^{-0.596t}\left[3715.62\cos(19.87t) + 108.75\sin(19.87t)\right]$

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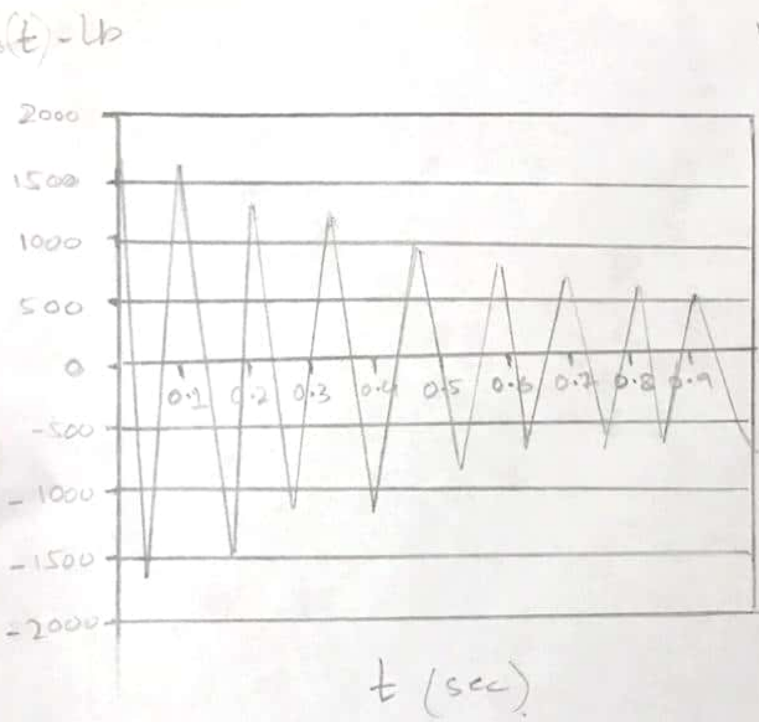

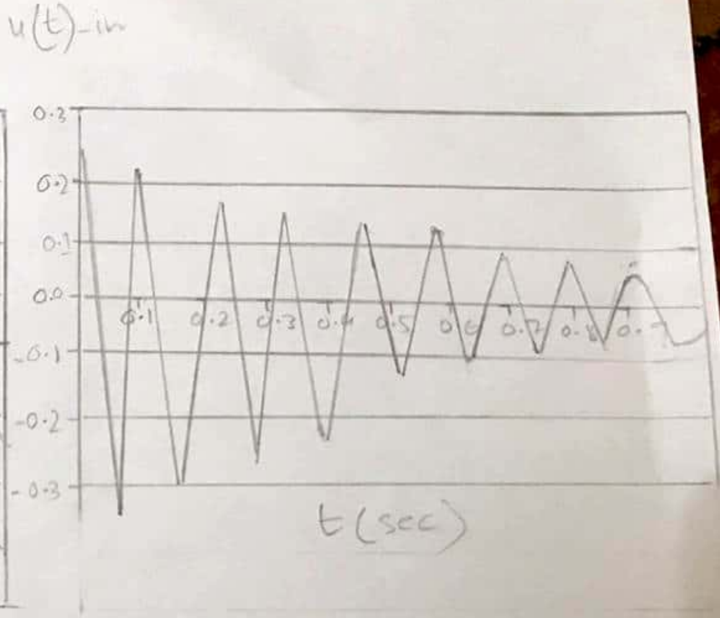

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(8)

Q#3

Given Data:-

Displacement of Tank = $\left(\frac{7386}{1000}\right) = 7.386$

Time taken to Complete 7 Cycles: 3.57 Sec

Amplitude of Displacement = 2.286
= 0.9

Required Data:-

(a) Damping Ratio

(b) Natural Ratio of Undamped Vibration

Sol:-

Displacement of Tank, $u_1 = 7.386$"

After 7 Cycles

i.e, After: $v = j = 7$, $u_{j+1} = u = 0.9$"

(a) Damping Ratio ζ = ?

$$\zeta = \frac{1}{2\pi j} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$\zeta = \frac{1}{2\pi 7} \ln\left[\frac{7.386}{0.9}\right]$$

$$\zeta = \frac{1}{2\pi \times 7} \ln\left[\frac{7.386}{0.9}\right]$$

$= 0.0463$

$= 4.63\%$

(b) Natural Period of Undamped Vibration

T_n : ?

As the 7 Cycles of Vibrations are Completed in 3.57 Sec.

⇒ Time required to Complete one Cycle, $T_D = \frac{3.57}{7} = 0.51$ Sec

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{(\omega_n \sqrt{1-\zeta^2})}$$

$$T_D = \frac{T_n}{\sqrt{(1-\zeta)^2}}$$

$$T_n = T_D \times \sqrt{1-\zeta^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.0463)^2}$$

$$T_n = 0.5093$$

$$T_n = 0.51 \text{ Sec}$$

(c) STIFFNESS:-

$$k = \frac{60 \times \cos 60^\circ}{7.386}$$

$$k = 4.06 \text{ k/in}$$

$$k = 4.06 \times n \times 100$$

$$k = = 4872 \text{ lb/ft}$$

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(d) weight of Tank, w = ?

$$w_n = \sqrt{k/m} = \sqrt{\frac{k}{w/g}} = \sqrt{\frac{k \cdot g}{w}}$$

$$w_n^2 = \frac{k \times g}{w}$$

$$w = \frac{k \times g}{w_n^2}$$

As $w_n = \frac{2\pi}{T_n}$

$$w = \frac{kg}{\left[\frac{4\pi^2}{T_n^2}\right]}$$

$$= \frac{kg \times T_n^2}{4\pi^2}$$

$$= \left(\frac{487216}{ft} \times \frac{32.2\ ft}{Sec^2}\right) \times \frac{(0.51\ Sec)^2}{4\pi^2}$$

$$w = 1033.57\ lb = 1.03k$$

(e) Damping Co-efficient, C = ?

It's known that

$$\zeta = \frac{c}{2mw_n}$$

$$= \zeta \times 2m \times w_n$$

$$= \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$= \frac{0.0463 \times 4 \times \pi \times \left(\frac{9953.93}{32.2}\right)}{0.51}$$

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(f) Number of Cycles to Reduce the displacement amplitude to 0.5" J=?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_i}{u_{j+1}}\right]$$

$$j = \frac{1}{2\pi \times 0.0463} \ln\left[\frac{7.386}{0.5}\right]$$

$$j = 9.24 \text{ or } 10 \text{ cycles}$$

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