Diffierential Equations :-

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Section :- A

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Question No ⇒ 1

(Part → 1)

Solution:-

$$w = \sin(x+ct) + \cos(2x+2ct)$$

$$\frac{\partial w}{\partial t} = \cos(x+ct) + c - \sin(2x+2ct) + 2c$$

$$\frac{\partial^2 w}{\partial t^2} = -\sin(x+ct) + c^2 - \cos(2x+2ct) + 4c^2 \longrightarrow (1)$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct)+2$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

$$= \left[-\sin(x+ct) - 4\cdot\cos(x+2ct)\right]$$

$$\boxed{c^2 \cdot \partial^2 w / \partial x^2}$$ Ans.

(Part → b)

Solution :.

Given Data: -

$$w = \tan(2x + ct)$$

Required :-

To check if it the solution of given eq or not

Solution:

$$w = \tan(2x + ct)$$

Partial diff w.r.to "x"

Now

$$\frac{\partial w}{\partial t} = c\sec^2(2x + ct)$$

again Diff;

$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left(c\sec^2(2x+ct)\right)$$

$$= 2c^2 \cdot 2\sec^2(2x+ct)\tan(2x+ct)$$

$$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$$

and

$$\frac{\partial^2 w}{\partial x^2} = 4\sec^2(2x+ct)\tan(2x+ct)$$

$$\Rightarrow 4c^2\sec^2(2x+ct)\tan(2x+ct)$$

$$\Rightarrow 4c^2\sec^2(2x+ct)\tan(2x+ct)$$

$$\Rightarrow \quad 0 = 0$$

satasified.

Hence $w = \tan(2x+ct)$ is the solution of given eq.

Question No → 2.

Solution :- Given function is

$$F(x) = \begin{cases} x; & -\pi < x \le 0 \\ 2x; & 0 \le x \le \pi \end{cases}$$

we have to find the fourier co-efficients a_0, a_n and b_n.

Now :-

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\cancel{\pi}}\left[0 - \frac{\pi^{\cancel{2}}}{2}\right] + \frac{\cancel{2}}{\cancel{\pi}}\left[\frac{\pi^{\cancel{2}}}{\cancel{2}} - 0\right]$$

$$\boxed{a_0 = -\frac{\pi}{2} + \pi \Rightarrow \frac{\pi}{2}} \rightarrow (a)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos nx\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0}(x\cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi}(2x\cos nx)\,dx$$

$$a_n = \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$a_n = \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

So

$$\boxed{a_n = \begin{cases} \frac{-2}{\pi n^2} \; ; & \text{if } n \text{ is odd} \\ 0 \; ; & \text{if } n \text{ is even} \end{cases}} \rightarrow (b)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\,dx$$

$$b_n = \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$\boxed{b_n = \frac{1}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right] = \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}} \rightarrow (c)$$

So the required series is : $F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2n-1)x}{(2n-1)^2} + 3\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}$$

Question No → 3

Solution :-

$$y'' - 4y' + 13y = 8\sin 3x \rightarrow (1)$$

$$y(0) = 1$$

$$y'(0) = 2$$

Associated Homogenous eq of (1) is

$$y'' - 4(y') + 13y = 0 \longrightarrow (2)$$

Change (2) into Auxiliary equation:-

Put $y = m$ in (2)

$$m^2 - 4m + 13 = 0$$

used Quadratic formula.

$$a = 1, \quad b = -4, \quad c = 13$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(4) \pm \sqrt{(-4)^2 - 4(1)(3)}}{2(1)}$$

$$m = \frac{4 \pm \sqrt{16 - 52}}{2}$$

P.T.O

$$m = \frac{4 \pm \sqrt{-36}}{2}$$

$$m = 4 \pm \sqrt{36 i}$$

$$m = \frac{4 \pm 6i}{2}$$

$$= 2 \pm 3i$$

$$\boxed{\begin{aligned} m_1 &= 2 + 3i \\ m_2 &= 2 - 3i \end{aligned}}$$

$$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \rightarrow (A)$$

Let

$$y_p = A \cos 3x + B \sin 3x \longrightarrow (*)$$

Diff :- w.r. to 'x'

$$y_p' = -3A \sin 3x + 3B \cos 3x$$

Again Diff :- w.r.to 'x'.

$$y_p'' = -9A \cos 3x - 9B \sin 3x.$$

Put in (1)

P.T.O

$$\Rightarrow (-9A\cos 3x - 9B\sin 3x)$$
$$- 4(-3A\sin 3x + 3B\cos 3x)$$
$$+ 13(A\cos 3x + B\sin 3x) = 8\sin 3x$$

$$\Rightarrow -9A\cos 3x - 12B\cos 3x + 13A\cos 3x$$
$$- 9B\sin 3x + 12A\sin 3x + 13B\sin 3x$$
$$= 8\sin 3x$$

$$\Rightarrow (-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)$$
$$\sin 3x = 8\sin 3x$$

$$\Rightarrow (4A - 12B)\cos 3x + (4B + 12A)$$
$$\sin 3x = 8\sin 3x$$

comparing co-effints

$\sin 3x \Rightarrow \boxed{4B + 12A = 8} \rightarrow$ (a)

$\cos 3x \Rightarrow 4A - 12B = 0 \Rightarrow 4A = 12B$

$= \boxed{A = 3B} \rightarrow$ (b)

Put (b) in (a)

$$4B + 12(3B) = 8$$
$$4B + 36B = 8$$
$$40B = 8$$

$\boxed{B = \frac{1}{5}} \rightarrow$ (c)

Put (c) in (b)

$\boxed{A = 3/5} \rightarrow$ (d)

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Put 'c' and 'a' in X

$$y_p = \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow B$$

The G. sol is

$$y = y_c + y_p$$

$$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow C$$

Now we need to find the values of c_1 and c_2 for This

Put $x = 0$ and $y = 1$ in C

$$1 = e^{2(0)}(c_1 \cos 3(0) + c_2 \sin 3(0)) + \frac{3}{5}\cos 3(0) + \frac{1}{5}\sin 3(0)$$

$$1 = (c_1(1) + 2(0)) + \frac{3}{5}(1) + \frac{1}{5}(0)$$

$$1 = c_1 + 3/5$$

$$c_1 = 1 - 3/5$$

$$\boxed{c_1 = \frac{2}{5}} \rightarrow (xx)$$

Diff : C w.r.to 'x'

$$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$$

P.T.O

$- \frac{6}{5} \sin + \frac{3}{5} \cos 3x \longrightarrow$ (D)

Put $y' = 2, \quad x = 0 \quad$ in (D)

$$y' = c_1 (2e^{2x} \cos 3x - 3e^{2x} \sin 3x)$$
$$+ c_2 (2e^{2x} \sin 3x + 3e^{2x} \cos 3x)$$
$$- \frac{6}{5} \sin 3x + \frac{3}{5} \cos 3x$$

Put - $y' = 2, \quad x = 0$

$$2 = c_1 (2e^{2(0)} \cos 3(0) - 3e^{2(0)} \cos 3(0))$$
$$+ c_2 (2e^{2(0)} \sin 3(0) + 3e^{2(0)} \cos 3(0))$$
$$- \frac{6}{5} \sin 3(0) + \frac{3}{5} \cos 3(0)$$

$$2 = c_1 (2) + c_2 (3) - 0 + 3/5$$
$$2 = 2c_1 + 3c_2 + 3/5$$

Put $c_1 = 2/5$

$$2 = 4/5 + 3c_2 + 3/5$$

$$2 = 7/5 + 3c_2$$

$$3c_2 = 2 - 7/5$$

$$3c_2 = 3/5$$

$$\boxed{c_2 = 3/15} \longrightarrow \text{xxx}$$

P-T-O

Put xx and xxx in C

$$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$$

$$\boxed{y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x}$$

↳ Required General solution :-

Question NO → 04

Solution :-

Solve

$$(D^2 - DD')\,z = \cos x \cos 2y \rightarrow (1)$$

As it is already in symbolical form as

$$(D^2 - DD')\,Z = \cos x \cos 2y$$

Put A.E

$$D^2 - DD' = 0$$

$$\Rightarrow \quad m^2 - m = 0 \qquad \because \frac{D}{D'} = m \text{ i.e, } D = n,\ D' = 1$$

$$\Rightarrow \quad m = 0, 1$$

$$\therefore \quad C.F = f_1(y) + f_2(y+x)$$

From eq (1)

$$P.I = \frac{1}{D^2 - DD'} \cos x \cos 2y$$

$$= \frac{1}{2} \cdot \frac{1}{D^2 - DD'} 2 \cos x \cos 2y$$

$$\because 2 \cos A \cos B = \cos(A+B) + \cos(A-B).$$

$$C.F = f_1(y-x) + x f_2(y-x)$$

$$P.I = \frac{1}{D^2 + 2DD' + D'^2}\left[2(y-x) + \sin(x-y)\right]$$

$$= \frac{1}{(D+D')^2}\left[2(y-x) + \sin(x-y)\right]$$

General Method here $m = -1$, $y - x = c$

$$= \frac{1}{D+D'} \int \left[2c + \sin(-c)\right] dx$$

$$= \frac{1}{D+D'}\left[2cx - (\sin c)x\right]$$

Replacing "c" by $y - x$.

$$\Rightarrow \frac{1}{D + D'} \left[2x(y-x) - x \sin(y-x) \right]$$

Again $m = -1$

Put $y - x = c$

$$= \int [2xc - x \sin c)\, dx = cx^2 - \frac{x^2}{2} \sin c$$

Replace "c" by "y - x"

$$= x^2(y-x) - \frac{x^2}{2} \sin(y-x) = x^2 y - x^3 + \frac{x^2}{2} \sin(x-y)$$

There fore the required solution of (1) is given by.

$$z = CF + P.I = f_1(y-x) + x f_2(y-x) + x^2 y - x^3 + \frac{1}{2} x^2 \sin(x-y).$$

Ans.