Major Assignment Spring 2020

LINEAR ALGEBRA

Total Marks :20

Submitted to :

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ID = 6844
BS (SE) Section B (8th semester)

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Q 1.- Compute adjoint of

(i) $A = \begin{bmatrix} 1 & 2 & \text{2nd ID} \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

2nd ID - 2nd Number of your ID

SOLUTION :-

My ID is = $\boxed{6844}$

2nd ID is Number is 8

$$A = \begin{bmatrix} 1 & 2 & 8 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\text{Adj}(A) = \text{Adj} \begin{bmatrix} 1 & 2 & 8 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\text{Adj}(A) = \begin{bmatrix} +\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} & -\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} & +\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix} \\ -\begin{bmatrix} 2 & 8 \\ 1 & 2 \end{bmatrix} & +\begin{bmatrix} 1 & 8 \\ 3 & 2 \end{bmatrix} & -\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \\ +\begin{bmatrix} 2 & 8 \\ 3 & 1 \end{bmatrix} & -\begin{bmatrix} 1 & 8 \\ 2 & 1 \end{bmatrix} & +\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \end{bmatrix}$$

$$\text{Adj}(A) = \begin{bmatrix} +(3\times2-1\times1) & -(2\times2-1\times3) & +(2\times1-3\times3) \\ -(2\times2-8\times1) & +(1\times2-8\times3) & -(1\times1-2\times3) \\ +(2\times1-8\times3) & -(1\times1-8\times2) & +(1\times3-2\times2) \end{bmatrix}$$

$$\text{Adj}(A) = \begin{bmatrix} +(6-1) & -(4-3) & +(2-9) \\ -(4-8) & +(2-24) & -(1-6) \\ +(2-24) & -(1-16) & +(3-4) \end{bmatrix}$$

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$$Adj(A) = \begin{bmatrix} 5 & -1 & -7 \\ 4 & -22 & 5 \\ -22 & 15 & -1 \end{bmatrix}$$

$$Adj(A) = \begin{bmatrix} 5 & 4 & -22 \\ -1 & -22 & 15 \\ -7 & 5 & -1 \end{bmatrix}$$

So Adj(A) Answer is

$$\boxed{Adj(A)} = \begin{bmatrix} 5 & 4 & -22 \\ -1 & -22 & 15 \\ -7 & 5 & -1 \end{bmatrix} \rightarrow \text{Answer}$$

ii.

$$B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$$

Adjoint of B

Solution :→

$$B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$$

$$Adj(B) = Adj \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} +\begin{bmatrix} -1 & 8 \\ -2 & 8 \end{bmatrix} & -\begin{bmatrix} 2 & 8 \\ 5 & 8 \end{bmatrix} & +\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix} \\ -\begin{bmatrix} 4 & 5 \\ -2 & 8 \end{bmatrix} & +\begin{bmatrix} 3 & 5 \\ 5 & 8 \end{bmatrix} & -\begin{bmatrix} 3 & 4 \\ 5 & -2 \end{bmatrix} \\ +\begin{bmatrix} 4 & 5 \\ -1 & 8 \end{bmatrix} & -\begin{bmatrix} 3 & 5 \\ 2 & 8 \end{bmatrix} & +\begin{bmatrix} 3 & 4 \\ 2 & -1 \end{bmatrix} \end{bmatrix}$$

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$$= \begin{bmatrix} +(-1\times8-8\times(-2)) & -(2\times8-8\times5) & +(2\times(-2)-(-1)\times5) \\ -(4\times8-5(-2)) & +(3\times8-5\times5) & -(3\times(-2)-4\times5) \\ +(4\times8-5\times(-1)) & -(3\times8-5\times2) & +(3\times(-1)-4\times2) \end{bmatrix}$$

$$= \begin{bmatrix} +(-8+16) & -(16-40) & +(-4+5) \\ -(32+10) & +(24-25) & -(-6-20) \\ +(32+5) & -(24-10) & +(-3-8) \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 24 & 1 \\ -42 & -1 & 26 \\ 37 & -14 & -11 \end{bmatrix}$$

$$\text{Adj}(B) = \begin{bmatrix} 8 & -42 & 37 \\ 24 & -1 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

So Adj (B) Answer is

$$\text{Adj } B = \begin{bmatrix} 8 & -42 & 37 \\ 24 & -1 & -14 \\ 1 & 26 & -11 \end{bmatrix} \longrightarrow \text{Answer}$$

Question No 1

Completed.

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Question No 2 :→
Find The cofactors of A_{21}, A_{31}, A_{33} if

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

SOLUTION :→

$$\text{Cofactor}(A) = \text{Cofactor}\begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

We find That

A_{21}, A_{31}, A_{33}

Cofactor of A_{21}

$$\text{cofactor of } -2 = A_{21} = -\begin{bmatrix} -2 & 3 \\ -3 & 2 \end{bmatrix}$$

$$A_{21} = -(-2 \times 2 - 3 \times (-3))$$

$$= -(-4+9)$$

$$A_{21} = -5$$

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Cofactor of A_{31}

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

$$\text{Cofactor of } 4 = A_{31} = + \begin{bmatrix} -2 & 3 \\ 3 & 1 \end{bmatrix}$$

$$= +(2 \times 1 - 3 \times 3)$$

$$= +(-2-9)$$

$$A_{31} = -11$$

Cofactor of A_{33}

$$\text{cofactor of } 2 = A_{33} = + \begin{bmatrix} 1 & -2 \\ -2 & 3 \end{bmatrix}$$

$$= +(1 \times 3 - (2) \times (-2))$$

$$= +(3-4)$$

$$A_{33} = -1$$

So that cofactor of A_{21}, A_{31}, A_{33} is $= -5, -11, -1$.

Answer

The cofactor of A is

$$\text{cofactor } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ -5 & A_{22} & A_{23} \\ -11 & A_{32} & -1 \end{bmatrix}$$

Answer

Q 2 is Completed.

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Question No : 3

Find Eigen Values & Eigen Vectors of

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \quad \& \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution :→

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$$

First we

Find Eigen Values

$$|A - \lambda| = 0$$

$$\begin{vmatrix} (2-\lambda) & 1 & 1 \\ 1 & (3-\lambda) & 2 \\ -1 & 1 & (2-\lambda \end{vmatrix} = 0$$

$$= \cancel{(2-\lambda)(3-\lambda)\times(2-\lambda)-2\times1)\ -1(\ 1-\lambda}$$

$$= (2-\lambda)(3-\lambda)\times(2-\lambda)-2\times1)-1(1\times(2-\lambda)-2\times(-1))+1((1\times1)-(3-\lambda)\times(-1))=0$$

$$= (2-\lambda)(((6-5\lambda+\lambda^2)-2)-1(2-\lambda)-(-2))+1(1-(-3+\lambda)=0$$

$$= -(2-\lambda)(4-5\lambda+\lambda^2)-1(4-\lambda)+1(4-\lambda)=0$$

$$= (8-14\lambda+7\lambda^2)-(4-\lambda)+(40-\lambda)=0$$

$$= (-\lambda^3+7\lambda^2-14\lambda+8)=0$$

$$\cancel{= -(\lambda^3+7x^2-}$$

$$= -(\lambda-1)(\lambda-2)(\lambda-4)=0$$

$$= (\lambda-1)=0 \text{ or } (\lambda-2)=0 \text{ or } (\lambda-4)=0$$

The eigen values of The Matrix A are given by $\boxed{\lambda = 1, 2, 4}$

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① Eigen Vectors for $\lambda = 1$

$\lambda = 1$ is That

$$|A - \lambda I| = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - 1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ -1 & 1 & 1 \end{bmatrix}$$

Now Reduce This Matrix

$$R_2 \leftarrow R_2 - R_1$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$R_3 = R_3 + R_1 \rightarrow = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$$

Interchanging Rows $R_2 \leftrightarrow R_3$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$R_2 \leftarrow R_3 \div 2 \longrightarrow = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$R_1 \leftarrow R_1 - R_2 \longrightarrow = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$R_3 \leftarrow R_3 - R_2 \longrightarrow = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

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The system associated with the eigen values $\lambda = 1$

$$|A - 1\lambda| \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 = 0, \quad x_2 + x_3 = 0$

$\Rightarrow x_1 = 0, \quad x_2 = -x_3$

$\Rightarrow$ eigen vactors corresponding to the eigen values $\lambda = 1$ is

$$V = \begin{bmatrix} 0 \\ -x_3 \\ x_3 \end{bmatrix} \qquad \text{Let } x_3 = 1$$

$$V_1 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

② Eigen Vactors for $\lambda = 2$

$$|A - \lambda| = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & 0 \end{bmatrix}$$

Now Reduce This matrix interchanging rows $R_1 \Leftrightarrow R_2$

$$= \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

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$R_3 \leftarrow R_3 + R_1$

$$= \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$$

interchanging rows $R_2 \leftrightarrow R_3 = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$

$\Rightarrow R_2 \leftarrow R_2 \div 2 \Rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$\Rightarrow R_1 \leftarrow R_1 - R_2 \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

The system associated with the eigen vectors $\lambda = 2$

$$(A - 2I)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 + x_3 = 0, \quad x_2 + x_3 = 0$

$\Rightarrow x_1 = -x_3, \quad x_2 = -x_3$

$\longrightarrow$ eigen vectors corresponding to the eigenvalue $\lambda = 2$ is

$$\underline{v} = \begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \end{bmatrix}$$

Let $x_3 = 1$

$$\underline{v_2} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

③ Eigen Vectors for $\lambda = 4$

$$A - \lambda I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 2 \\ -1 & 1 & -2 \end{bmatrix}$$

Now Reduce This Matrix $\Rightarrow \begin{bmatrix} 1 & -0.5 & -0.5 \\ 1 & -1 & 2 \\ -1 & 1 & -2 \end{bmatrix}$

$R_2 \leftarrow R_2 - R_1$

$$= \begin{bmatrix} 1 & -0.5 & -0.5 \\ 0 & -0.5 & 2.5 \\ -1 & 1 & -2 \end{bmatrix}$$

$$R_3 \leftarrow R_3 + R_1 = \begin{bmatrix} 1 & -0.5 & -0.5 \\ 0 & -0.5 & 2.5 \\ -1 & 1 & -2 \end{bmatrix}$$

$$R_3 \leftarrow R_3 - 0.5 \times R_2 \longrightarrow \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

The system associated with the eigen Values of $\lambda = 4$

$$(A - 4I) \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow x_1 - 3x_3 = 0 \quad , \quad x_2 - 5x_3 = 0$

$\Rightarrow x_1 = 3x_3 \quad , \quad x_2 = 5x_3$

eigen Vectors Corresponding to the eigen value $\lambda = 4$ is

$$\underline{v} = \begin{bmatrix} 3x_3 \\ 5x_3 \\ x_3 \end{bmatrix}$$

Let $x_3 = 1$

$$= \underline{V_3} = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$$

Part 1 Completed.

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(ii)

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

First we find Eigen Values of

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so

$$|A - \lambda I| = 0$$

$$= \begin{vmatrix} (1-\lambda) & 0 & 0 \\ 0 & (1-\lambda) & 0 \\ 0 & 0 & (1-\lambda) \end{vmatrix} = 0$$

$$= (1-\lambda)((1-\lambda)\times(1-\lambda) - 0\times 0) - 0(0\times(1-\lambda) - 0\times 0) + 0(0\times 0 - (1-\lambda)) = 0$$

$$= (1-\lambda)((1-2\lambda+\lambda^2)-0) - 0(0-0) + 0(0-0) = 0$$

$$= (1-\lambda)(1-2\lambda+\lambda^2) - 0(0) + 0(0) = 0$$

$$= (1-3\lambda+3\lambda^2-\lambda^3) - 0 + 0 = 0$$

$$= (-\lambda^3+3\lambda^2-3\lambda+1) = 0$$

$$= -(\lambda-1)(\lambda-1)(\lambda-1) = 0$$

$$= (\lambda-1) = 0 \quad \text{or} \quad (\lambda-1) = 0 \quad \text{or} \quad (\lambda-1) = 0$$

so

The eigen Values of The matrix (I) are given by $\boxed{\lambda - 1}$

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(1) Eigen Vectors for

$$\boxed{\lambda = 1}$$

$$= A - \lambda I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now Reduce This Matrix

The system associated with The eigenValue $\lambda = 1$

$$(A - \lambda I)\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

eigen vectors Corresponding to The eigen Value $\lambda = 1$ is

$$v = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Let $x_3 = 1$, $x_1 = 0$, $x_2 = 0$

$$\underline{v_1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Let $x_1 = 0$, $x_2 = 0$

$$\underline{v_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Let $x_1 = 0$, $x_2 = 0$

$$\underline{v_3} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Completed

End of Assignment.