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Section A

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Subject Introduction to structural Dynamics and Earthquake Engg

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QNo 1

Sol The general E.O.M for SDOF system is

$$Ku + c\dot{u} + m\ddot{u} = p(t)$$

In our case system is undamped $(c=0)$ undergoing free vibration $(p(t)=0)$

Hence general EOM become $Ku + m\ddot{u} = 0$

$$K = 3EI/L^3$$

$$K = \frac{3 \times 29000 \; K/in^2 \times 150 \, in^4}{(10 \times 12 \, in)^3}$$

$$\boxed{K = 7.55 \; K/in}$$

In order to eliminate the chances of mistake during calculation it is more appropriate to use fundamental units like lb, ft sec or kg m, Sec

$K = 7.55 \text{ K/in} = 90625 \text{ lb/ft}$

$$m = \frac{7698 \text{ lb}}{32.2 \text{ ft/sec}^2}$$

$$\boxed{m = 239.068 \text{ slug}}$$

$$W_n = \sqrt{\frac{K}{m}}$$

$$W_n = \sqrt{\frac{90625}{239.068}}$$

$$\boxed{W_n = 19.469 \text{ rad/sec}}$$

$$T_n = \frac{2\pi}{W_n} \quad \frac{2\pi}{19.469}$$

$$\boxed{T_n = 0.322 \text{ sec}}$$

Substituting the corresponding value in eq (1)

$$90625\, \ddot{u} + 239.068\, \ddot{u} = 0$$

where "K" is in lb/ft and 'm' is in lb sec/ft^2

General solution to the EOM for undamped free vibration is

$u(t) = u(0)\cos(\omega_n t)$ is $\dot{u}(0)/\omega_n \sin \omega_n t$

$u(0) = \frac{1}{2}'' = \frac{1}{24}$ ft and $\dot{u}(0) = 0$

$u_t = ((\frac{1}{24}) \times \cos(19.46t) + 0)$

$u(t) = \frac{1}{24} \times \cos(19.46t)$

Equivalent static force at any time "t" is

$f_s(t) = K \cdot u(t)$

$$f_s(t) = \frac{90625 \times \cos(19.46t)}{24}$$

$f_s(t) = 3776.04 \cos(19.46t)$

Amplitude of dynamic displacement u_0 for undamped free vibration

$$u_0 = \sqrt{(u(0))^2 + (\dot{u}(0)/\omega_n)^2}$$

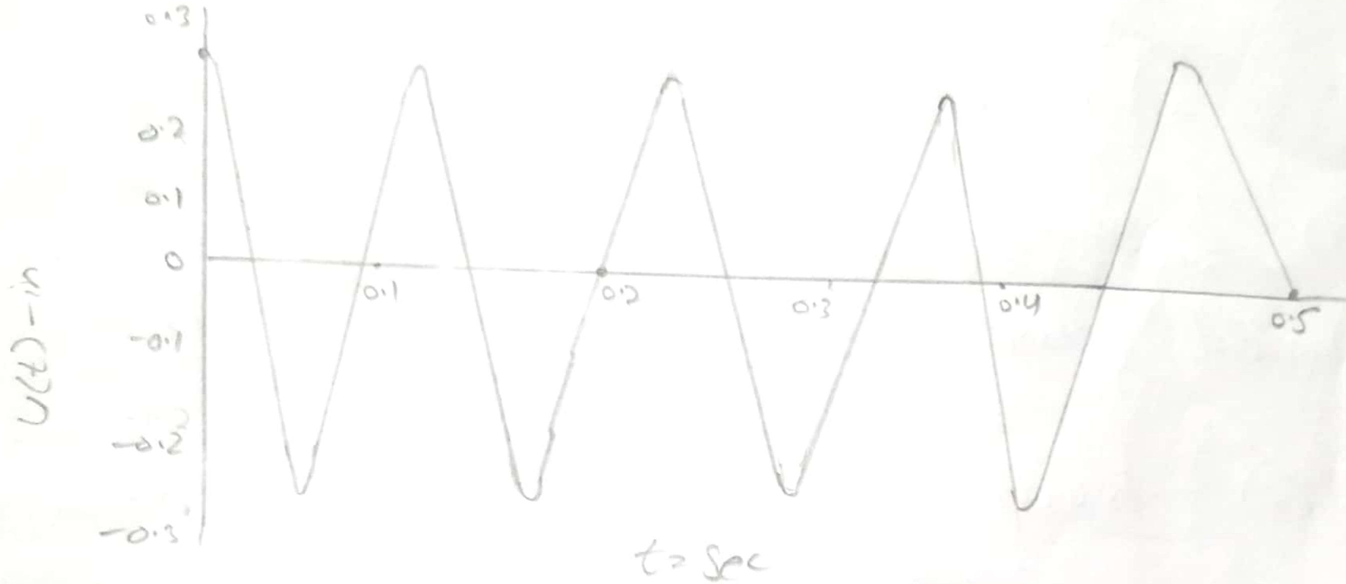
$$u_0 = \sqrt{((\tfrac{1}{24})^2 + 0}$$

$$\boxed{u_0 = \frac{1}{24} \text{ ft}}$$

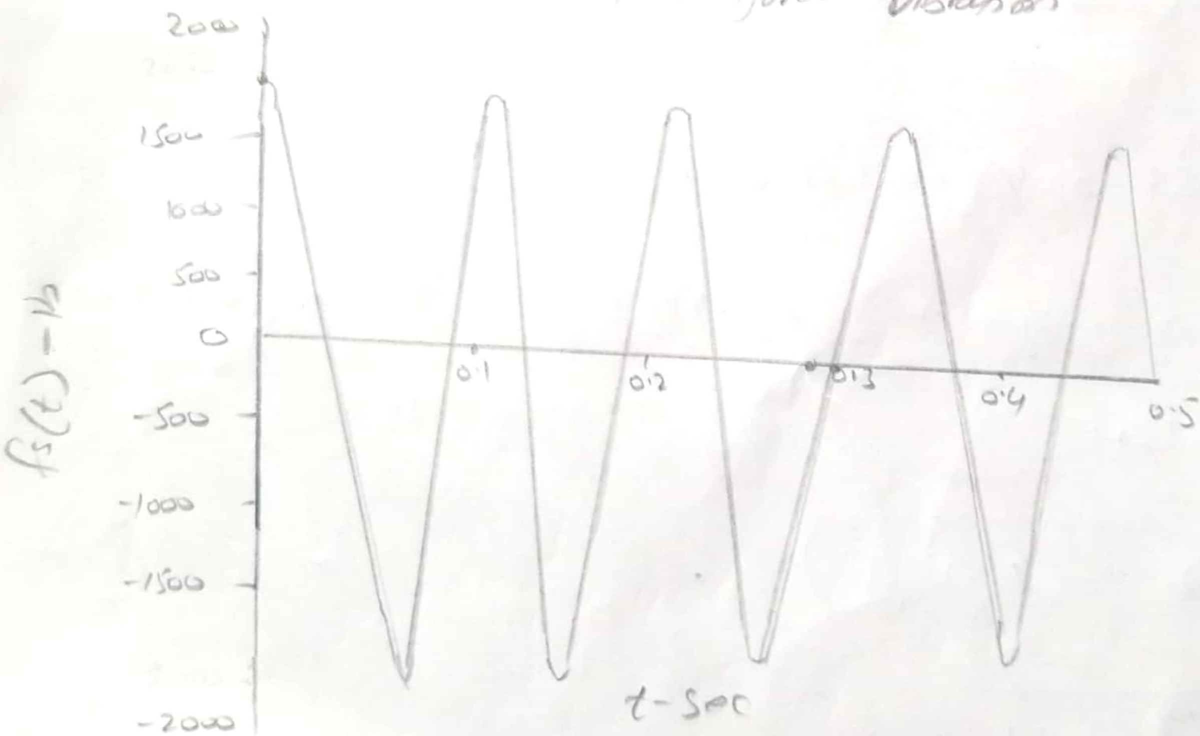
Amplitude of equivalent static force.

$$Ku_0 = 90625 \times \frac{1}{24}$$

$$\boxed{Ku_0 = 3776.04\ lb}$$



undamped force vibration



undamped free vibration

(Q No = 02)

Solution

(769813)

E.O.M for damped free vibration is

$$Ku + c\dot{u} + m\ddot{u} = 0 \quad \text{----- (i)}$$

it is know from (Question 1)

$K = 90625$ lb/ft and $m = 239.068$ Slug

$W_n = 19.469$ rad/sec $\quad m = 239.068$ lb.sec²/ft

$C = \xi \times 2m\omega_n = 2 \times 239.068 \times 19.469 \times \xi$

($\xi = 0.03 - 0.05$ with considreble cracking the damping ratio)

$C = 2 \times 239.068 \times 19.469 \times 0.05$

$\boxed{C = 465.441}$ lb.Sec/ft

By substituting values of K, C and m in eq (i) we get

$$90625u + 465.441\dot{u} + 239.068\ddot{u} = 0$$

⇒ Solution to the E.O.M for damped free vibration is

$$u(t) = e^{-\xi w_n t}\left(u(0)\cos(w_D t) + \frac{1}{w_D}(\dot{u}(0)) + u(0)\,\xi w_n \sin(w_D t)\right)$$

$$w_D = 19.469 \text{ rad/sec}$$

$$u(t) = e^{-0.05 \times 19.469t}\left(\frac{1}{24} \times \cos(19.469t) + \frac{1}{19.469} * (0 + \frac{1}{24} \times 0.05 \times 19.469 \times \sin(19.469)\right)$$

$$u(t) = e^{-0.973t}\left(0.0417 \times \cos(19.469t) + 0.051 \times 0.0417 \times \sin(19.469t)\right)$$

$$u\,t = e^{-0.973t}\left(0.0417 \times \cos(19.469t) + 0.0021 \times \sin(19.469t)\right)$$

$$f_s(t) = K \cdot u(t) = 90625 \times u(t)$$

$$f_s(t) = e^{-1373t}(3779.1\cos(19.469t) + 1903.12 \times \sin(19.469t))$$

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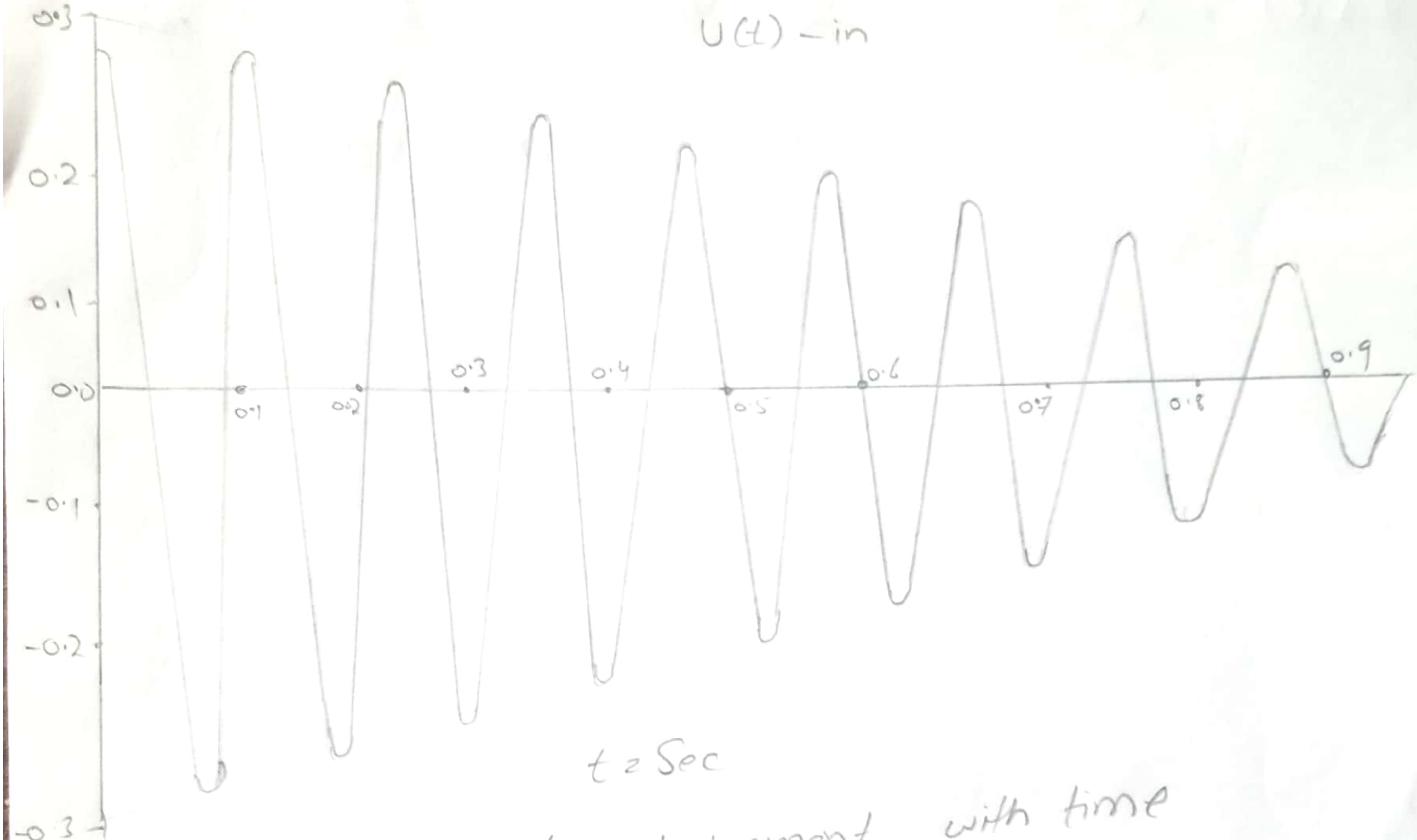
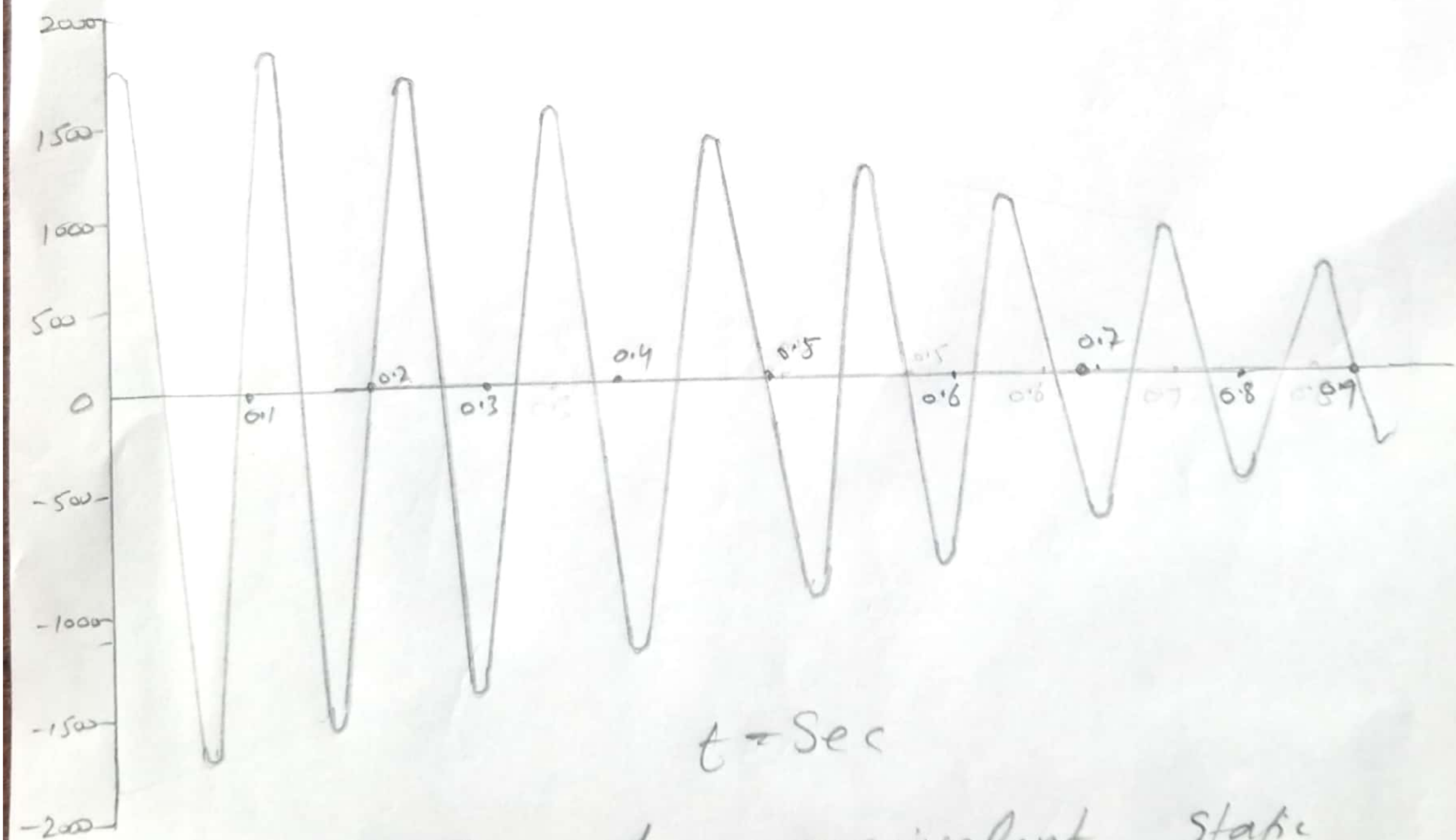


variation of displacement with time



variation of equivalent static force with time.

(1)

Q No = 3

Solution

$$U_1 = \frac{7698}{1000} = 7.698 = \boxed{7.7''}$$

After $J = 7$, $U_{J+1} = U_6 = 2.286\text{cm} = 0.9''$

(a) ξ = Damping ratio = ?

$$J = \frac{1}{2\pi\xi} \ln\left(\frac{U_1}{U_{J+1}}\right)$$

$$7 = \frac{1}{2\pi\xi} \ln(7.7/0.9)$$

$$\xi = 0.049 = \boxed{4.9\%}$$

(2)

(B) Natural period of undamped vibration.

$T_n = ?$

7 cycles of vibrations are completed

In 3.57 sec.

time required to complete one cycle $= t_D = \frac{3.57}{7}$

$$\boxed{T_D = 0.51 \text{ Sec}}$$

Now

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{\left(\omega_n \sqrt{1-\zeta^2}\right)}$$

$$\Rightarrow T_D = \frac{T_n}{\sqrt{1-\zeta^2}}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\zeta^2}$$

$$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.049)^2}$$

$$\Rightarrow T_n = 0.5094 = \boxed{0.51 \text{ sec}}$$

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(C) Stiffness of structures. $k = ?$

$$K = \frac{60 \times \cos 60^\circ}{7.7} = 3.9 \text{ K/in}$$

$$\boxed{K = 46800 \text{ lb/ft}}$$

(d) Weight of the tank $W = ?$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{(W/g)}}$$

$$= \sqrt{\frac{K \times g}{W}}$$

$$\Rightarrow \omega_n^2 = K \times g / W$$

$$\Rightarrow W = K \times g / \omega_n^2$$

Also

$$\omega_n = 2\pi / T_n$$

$$W = Kg \left(\frac{4\pi^2}{T_n^2} \right) = Kg \times \frac{T_n^2}{4\pi^2}$$

$$W = \frac{46800 \times 32.2 \times (0.51)^2}{4\pi^2}$$

$W = 9928.5$ lb

$$\boxed{W = 9.93\ K}$$

(e) **Damping Coefficient**: $C = ?$

it is known that $\zeta = \frac{C}{2m\omega_n}$

$\Rightarrow C = \zeta \times 2m\omega_n = \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$

$\Rightarrow C = 0.649 \times 2 \times 2 \times \left(\frac{\pi}{0.51}\right)\left(\frac{9928.5}{32.2}\right)$

$$\boxed{C = 372.27\ \text{lb.sec/ft}}$$

(f) No of cycles to reduce displacement amplitude from 7.7 in to 0.5" $j = ?$

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$\Rightarrow j = \frac{1}{2\times\pi\times 0.049} \ln\left(\frac{7.7}{0.5}\right)$$

$$\Rightarrow \boxed{j = 8.69 \text{ or } 9 \text{ cycles.}}$$