Day. M T W T F S Page #01 Date: __/__/__

Subject: Differential equation

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Q No. (01) Find the Fourier Series representation of $f(t) = 1+t$, $-\pi \le t < \pi$.

Solution:

$$f(t) = 1 + t \quad -\pi \le t \le \pi$$

Here we use the Formula

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \quad (1)$$

$$= a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\,dt$$

$$= a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} (1+t)\,dt$$

$$= a_0 = \frac{1}{2\pi}\left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$= a_0 = \frac{1}{2\pi}\left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(\frac{-\pi^2}{2}\right)\right)$$

$$= a_0 = \frac{1}{2\pi}\left(2\pi + \frac{2\pi^2}{2}\right)$$

$$= \boxed{a_0 = \frac{1}{2\pi}\left(2\pi + \pi^2\right)}$$

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P#03

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$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)(\cos nt)\, dt$$

$$a_n = \frac{1}{\pi}\left.(1+t)\frac{\sin nt}{n}\right]_{-\pi}^{\pi} - \int\left(\frac{\sin nt}{n}\frac{d}{dt}(1+t)\right)$$

$$a_n = \frac{1}{\pi}\left((1+t)\frac{\sin n\pi}{n} - \left.\frac{\cos nt}{n^2}\right|_{-\pi}^{\pi}\right)$$

$$a_n = \frac{-1}{n^2\pi}(\cos n\pi - \cos n(-\pi))$$

$$a_n = \frac{-1}{n^2\pi}(-1-(-1))$$

$$\boxed{a_n = 0}$$

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$= b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\,dt$

$= b_n = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi}\sin nt - \int\left(\int \sin nt \frac{d}{dt}(1+t)\,dt\right)\right)$

$= b_n = \frac{1}{\pi}\left(\frac{(1+t)\cdot(-\cos nt)}{n}\Big|_{-\pi}^{\pi} - \int\left(\frac{-\cos nt}{n}(1)\right)\right)$

$= b_n = \frac{1}{\pi}\left(\frac{-(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} + \left(\frac{\cancel{\sin nx}^{\,0}}{n^2}\Big|_{-\pi}^{\pi}\right)\right)$

$= b_n = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - (1+(\pi))(\cos n(\pi)\right)$

$= b_n = \frac{1}{n\pi}\left(\cancel{\cos n\pi} + \pi\cos n\pi - \cancel{\cos \pi} + \pi\cos n\pi\right)$

$= b_n = \frac{-1}{n\cancel{\pi}}(2\cancel{\pi}\cos n\pi)$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$= \quad b_n = \frac{2}{n}(-1)^{n+1}$

so eq become

$$= \boxed{f(t) = \frac{1}{2\pi}(2\pi + \pi^2) + 0 + \sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n}\sin t}$$

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Q No. (02) Calculate the characteristic equation the Eigen values of the system where A is given by.

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$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Solution:

We have;

A= Given Matrix

$$(A - \lambda I)X = 0$$

Step (02):

We have the characteristic equation is given by.

$$= |A - \lambda I| = 0$$

$$= \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$= \begin{bmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{bmatrix} = 0$$

Step #03

$$\lambda^3 - \left|\text{sum of Diagonal element}\right| \lambda^2 + \left|\text{sum of Diagnol minors}\right| \lambda - |A| = 0 \quad \text{—(B)}$$

Sum of Diagonal elements $= 1+1+2 = 4$

Sum of Diagonal element $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (1)$

$= (-6 + 2 + 1)$

$= -3$

By putting values in Eq (B).

$= \lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0$ — (C)

$= |A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$

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$= 1(2-8) - 0 + 1(6-0)$

$= -6+6$

$= 0$

By putting values in (c)

$= \lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$

$= \lambda^3 - 4\lambda^2 - 3\lambda = 0$

$= \lambda(\lambda^2 - 4\lambda - 3) = 0$

$= \lambda = 0$

$= \lambda^2 - 4\lambda - 3 = 0$

Using quadratic formula.

$= \lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = 1$, $b = 4$, $c = -3$

$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$

$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$

$\lambda = \frac{4+\sqrt{28}}{2}$, $\lambda = \frac{4-\sqrt{28}}{2}$

we have Eigen value.

$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right)$ required answer.

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Day. M T W T F S P #(08) Date: __/__/__

Q No. (03) Solve the following system of linear equations:

$5x + 4z + 2m = 3$
$x - y + 2z + m = 1$
$4x + y + 2z = 2$
$x + y + z + m = 0$

Solution:

Write in Augmented Form

$$= \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] R_1 - R_2$$

$$= \left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 1 \\ 5 & 0 & 4 & 2 & 3 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] R_2 - 5R_1$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & 5 & -6 & -3 & -2 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] R_3 - 4R_1$$

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$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & -6 & -3 & -2 \\ 0 & 5 & -6 & -4 & -3 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

$R_3 + R_2$

$$= \left[\begin{array}{cccc|c} 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & -6 & -3 & -2 \\ 0 & 0 & -12 & -7 & -5 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] R_4 - R_1$$

$$= \left[\begin{array}{cccc|c} 1 & -1 & -2 & 1 & 1 \\ 0 & -5 & -6 & -3 & -2 \\ 0 & 0 & -12 & -7 & -5 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$R_4 + \frac{2}{5} R_2$

$$= \left[\begin{array}{cccc|c} 1 & 1 & -2 & 1 & 1 \\ 0 & -5 & -6 & -3 & -2 \\ 0 & 0 & -12 & -7 & -5 \\ 0 & 0 & -\frac{17}{5} & 6/3 & -7/5 \end{array}\right]$$

$R_3 \leftrightarrow R_4$

$$= \left[\begin{array}{cccc|c} 1 & 1 & -2 & 1 & 1 \\ 0 & -5 & -6 & -3 & -2 \\ 0 & 0 & -17/5 & 6/5 & -17 \\ 0 & 0 & -12 & -7 & -5 \end{array}\right] R_4 - 2R_3$$

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$$= \left[\begin{array}{cccc|c} 1 & 1 & -2 & 1 & 1 \\ 0 & -5 & -6 & -3 & 3 \\ 0 & 0 & -17/5 & 6/5 & -17/5 \\ 0 & 0 & 0 & -1 & 1 \end{array}\right]$$

$$-m = 1$$

$$= -\frac{17}{5}z + \frac{6}{5}m = -17/5$$

$$= -\frac{17}{5}z = -\frac{17}{5} - \frac{6}{5}$$

$$-\frac{17}{5}z = \frac{-23}{5}$$

$$= z = \frac{-23}{\cancel{5}} \times \frac{\cancel{5}}{17}$$

$$= \boxed{z = \frac{23}{17}}$$

$$= -5y - 6z - 3m = 3$$

$$= -5y \; 6\left(\frac{23}{17}\right) - 3 = 3$$

$$= -5y - \frac{138}{17} = 6$$

$$= -5y = \frac{17 \times 6 + 138}{17}$$

$$= \frac{102 + 138}{17}$$

$$y = \frac{\cancel{240}^{48}}{17 \times \cancel{5}}$$

$$\boxed{y = -48/17}$$

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$= x + y + 2z + m = 1$

$= x - \frac{48}{17} - 2\left(\frac{23}{17}\right) + 1 = 1$

$= x - \left(\frac{-48 - 46}{17}\right) = 0$

$= x = -2/17 = 0$

$$\boxed{x = \frac{2}{17}}$$

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Q No. (04)

Q NO. (04)

verify that
$u(x,t) = \sin(x+2t)$
is a solution of the one-dimensional wave equation.

Solution:

$$u(x,t) = \sin(x+2t)$$

= Differentiate w.r.t x.

$$= \frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \sin(x+2t)$$

$$= \frac{\partial u}{\partial x} = \cos(x+2t)\frac{\partial}{\partial x}(x+2t)$$

$$= \frac{\partial u}{\partial x} = \cos(x+2t)(1+0)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \cos(x+2t)$$

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$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)\frac{\partial}{\partial x}(x+2t)$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)}$$

and $u(x,t) = \sin(x+2t)$

Differentiate w.r.t "t"

$$\frac{du}{\partial t} = \frac{\partial}{\partial t}\sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t)\,(0+2)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial t} = (2) - \sin(x+2t)\,(0+2)$$

$$\boxed{\frac{\partial^2 u}{d t^2} = -4\sin(x+2t)}$$

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We know that,

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$-4\sin(x+2t) = c^2\left[-\sin(x+2t)\right].$$

$$-4\sin(x+2t) = -c^2\sin(x+2t)$$

$$-4\sin(x+2t) + c^2\sin(x+2t) = 0$$

For orbitary constant $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2\sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

It is verfied

For the orbitsary constant:

$c = 2$

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