Q1) Find

$$\int_0^1 \frac{4t^3-2t^2+2t-1}{2t^2+1}\, dl$$

Solution:~

$$\int_0^1 \frac{4t^3-2t^2+3t-1}{2t^2+1}\, dt$$

By Partial fraction method

Divide $4t^3-2t^2+3t-1$ by $2t^2+1$

$$\int_0^1 2t-1+\frac{t}{2t^2+1}\, dt$$

$$\int_0^1 2t\, dt + \int_0^1 -1\, dt + \int_0^1 \frac{t}{2t^2+1}\, dt$$

$$2\int_0^1 dt + \int_0^1 -1\, dt + \int_0^1 \frac{t}{2t^2+1}\, dt$$

using power ~~t~~ rule

$$2\left(\tfrac{1}{2}t^2\Big]_0^1\right) + \int_0^1 -1\, dt + \int_0^1 \frac{t}{2t^2+t}\, dt$$

combine ½ & $t^2$

$$2\left(\frac{t^2}{2}\Big]_0^1\right) + \int_0^1 -1\, dt + \int_0^1 \frac{t}{2t^2+1}\, dt$$

$$2\left(\frac{t^2}{2}\Big]_0^1\right) + (-t\Big]_0^1 + \int_0^1 \frac{t}{2t^2+1}\, dt$$

using substitution

let $u = 2t^2+1$ then $du = 4t\,dt$ so $\frac{1}{4}du = t\,dt$

$$= 2\left[\left[\frac{t^2}{2}\right]_0^1\right] + (-t)\Big]_0^1 + (-t)\Big]_0^1 + \int_1^3 \frac{1}{u}\cdot\frac{1}{4}\,du$$

$$= 2\left(\frac{t}{2}\right]_0^1\Big] + (-t)\Big]_0^1 + \int_1^3 \frac{1}{4u}\,du$$

Applying limit we get

$$\boxed{f(x) = 0.2746}$$

Q2) Find $\int_2^3 t \sin t^2\,dt$

Solution:~

let $u = t^2$

$du = 2t\,dt$

$dt = \frac{du}{2t}$

Replace the value of $t$ & $dt$

$$= \int_2^3 \cancel{t} \sin u \,\frac{du}{2\cancel{t}}$$

$$\int_2^3 \tfrac{1}{2} \sin u\,du$$

$$= -\tfrac{1}{2}\cos u \Big|_2^3$$

replace $u$ with $t^2$

$= -\frac{1}{2}\cos t^2 \Big|_2^3$ Applying limits $= -\frac{1}{2}\left(\cos(3)^2 - \cos(2)^2\right) = -\frac{1}{2}(\cos 9 - \cos 4)$

$= \boxed{0.0049}$ Ans